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SUBSIDENCE ANALYSIS OF HYDROELECTRIC DAM USING THE KALMAN FILTER – A CASE STUDY IN HOA BINH HYDROPOWER PLANT, VIETNAM

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Abstract. Hydroelectric dams have a great influence on the safety of the downstream area. Therefore, deformation monitoring for assessing the safety of dam should be carried out regularly. In order to improve efficiency of the dam management, it is necessary to analyse the displacement values in space, over time to assess overally the displacement of dam. In this purpose, an attempt was conducted to analyse the subsidence of hydroelectric dams located in Hoa Binh, Vietnam using one of the most useful method – Kalman filter. Kalman filter is the unique method that can determine influence of external factors (particularly, elevation of water level in the reservoir) on dams, simultaneously forecast the displacement values of dam in the future. Moreover, Kalman filter allows to predict subsidence accurately in about 6 months that is longer prediction time than other static models. These are clearly presented and discussed in the article. The obtained results demonstrate the high applicability of Kalman filter method in analysing and forecasting the subsidence of the Hoa Binh hydroelectric dam.

Keywords: subsidence monitoring, prediction, subsidence analysis, Kalman filter, hydroelectric dams, external factors.

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1. Introduction

Hydroelectric dams, especially the large rockfill dams with concrete face need to be supervises regularly because these structures are easy to happen deformation due to the causes of their load themselves and pressure of water in the reservoir. Therefore, deformation monitoring plays an important role to support evaluation of safety operation state of dams. However, the obtained results are only displacement values on surface, in order that supervision for dam is more efficient, the monitoring results have to be analyzed. After analyzing, it is able to assess overall displacement of dams (based on the discrete displacement value of each point), determine the influence of external factors on the displacement of dams and forecast the displacement in the future. In fact, deformation analysis and prediction have been cared, studied and applied at many dams in the world such as Shuibuya concrete-face rockfill dam in China (Zhou et al., 2011); earth dam in Chaozhou project (Wang et al., 2012); Alavian dam (Moradi & Ebrahimnezhed, 2017); Kouris earth dam in Cyprus (Pelecanos et al., 2018). With this necessity, a lot of methods for analyzing and predicting deformation of dams have been proposed and used. Artificial neural network theory developed long time ago and has been applied in analyzing and predicting dam deformation (Ma et al., 2009) especially the back propagation neural network model was suitably used for analysis of dam horizontal displacement (Jang et al., 2010). However, these documents also indicated that BPNN has some shortcomings that needs to be supported by several corrective measures or combining with other models such as SARIMA (the seasonal integrated auto-regressive moving average) or multi-regression to achieve the fitting forecast accuracy (Zou et al., 2018). Multiple linear regression model is the efficient one in analyzing and evaluating influence of external factors on dam, the calculation process is simple and easy (Liu et al., 2010). But, in order to achieve better fitting accuracy and prediction precision, the Error Correction model was proposed by Li et al. (2013). Other statistics methods were mentioned for analyzing subsidence of dam such as multivariate statistic, graph method… (Sigtryggsdottin et al., 2013, 2015), the authors proposed and applied but no compare and make a choice of the most suitable method. In order to improve the limitation, Sigtryggsdottin et al. (2018) conducted a study using the statistical model HST (Hydrostatic-Seasonal-Time) which considers the influence of load and time on the settlement of dam to analysis and predict. To improve poor performance in processing massive monitoring data due to the multidimensional data collinearity problem and simulating the inaccurate temperature field of the HST model, He and Li (2022) introduced the algorithm of BO (Bayesian optimization) combining with LGB (light gradient boosting tree). This algorithm can be applied well in modelling, high accuracy and efficiency in analyzing impact factors on displacement of dam.

Mentioned to assessing the influence of external factors on deformation of the dam (Bak, 2016) used the EDF model to analyze the influence of some factors such as rheological factors, elevation of water level in the reservoir.... EDF is one of the first statistical models established by Électricité de France to assess the state of arch dams. Later, the model was modified depending on the type of construction and the purpose of application. Although there are many different analytical methods, the EDF is the most suitable model for the analytical assumptions. The document (Pelecanos et al., 2018) also studied about evaluating the influence of the factors on the subsidence using the method of finite-element analysis. Factors were assessed including structure, water level in the reservoir and seismic activity. The obtained results shown that the water level in the reservoir affected settlement of the dam but it is smaller than the effect caused by the structure. Tran (2011) demonstrated the dependence of subsidence on the water level in the reservoir using single linear correlation analysis method. This study only determined relationship between subsidence of dam and the impact factor, not calculating how many the influence value was.

To conduct well two tasks of analysis: evaluation the influence of external factors and deformation prediction, some methods were proposed such as gene expression programming (GEP) and neurogenetics (NG) (Noorzad et al., 2014), input data consists of parameters: height of dams, factor of shape, void ratio, subsidence and time, the obtained forecasting results have high accuracy. These two methods are easy to implement and available applications on web, so the methods have great advantages in production practice. The other method is combination of the analysis of observation results, vector regression and small wave neural network that has a forecasting accuracy up to 99.19% (Luo et al., 2020), this method is implemented as follows: firstly, analyze the monitoring results by the EMD method (Empirical Mode Decomposition), determine the components affecting the subsidence, and then find a suitable forecasting model; next, using two methods SVR (Support Vector Regression) and WNN (Wavelet Neural Network) predict; finally, merge the forecasting results for the final result. Another new model that is used in three-dimensional displacement prediction (x, y, z) is the Wiener Discrete Process Acceleration Model (DWPAM) which was built on the basis of Kalman filter (Gamse & Oberguggenberger, 2016). This is a dynamic processing model whose advantage is eliminating the influence of physical properties on the displacement of construction. In addition, the model is also capable of detecting abnormal things to early warn, along with ensure safety for dam.

Kalman filter is a method that has been applied for analyzing and predicting for a long time (Lu, 2002; Lu & Li, 2013), many scientists researched application of Kalman filter and saw that this is efficient method, quick calculation, high prediction accuracy (Irughe et al., 2014; Dai et al., 2016). Moreover, this method can evaluate impact of factors on deformation of dam (Lu, 2003; Lu & Li, 2013). However, in almost articles, scope of research has only focused on horizontal displacement of dams, hardly on subsidence although subsidence also affects safety of dams Therefore, in this paper, subsidence of dam is mentioned and Kalman filter is applied for analyzing and predicting subsidence of dam in Vietnam.

Hoa Binh hydroelectric plant is one of the largest works that play important role in the energy security of the north of Vietnam. Hydroelectric dams stability is the key issue to maintaining the continuous operation of hydroeletric plant. In addition, dams usually have a great impact on the safety of the downstream area. Therefore, the major aim of this paper is to analyse the displacement values over time to determine influence of external factors on dams as well as forecast the displacement values of dam in the future. It would assist to improve the efficiency of data processing, which have a significant impact on management of the Hoa Binh hydroelectric dam. To do so, the analytical and forecasting method-Kalman filter was applied for the Hoa Binh dam. Influence of external factor (elevation of water level in the reservoir) on the dam and the subsidence prediction for this hydroelectric dam are presented and discussed.

2. Models of subsidence analysis

2.1. Overview of mathematical model establishment for analyzing and predicting subsidence

Assume that the subsidence model over time is expressed through a general function as follows:

$$S = f(t), \tag{1}$$

with the vector of parameters

$$Z = \left(z_1 \quad z_2 \quad \dots \quad z_k\right)^T. \tag{2}$$

Thus, rely on the results of displacement monitoring in n cycles to determine the parameter vector of the function (1). Symbolize the time and the obtained settlement values as vectors respectively: $T = (t_1 t_2 \dots t_n)^T$; $S = \left(S_1 \ S_2 \dots S_n\right)^T$. When the number of monitoring periods is larger than the number of parameters ($n > k$), the problem is solved according to the least squares principle, time (t) becomes the coefficient of the equation that includes k variables z_i to be determined, settlement (S) is the measured values vector.

The correction equation system has the form

$$V = AZ + L, \tag{3}$$

where vector of the free numbers $L = S - S^o$.

The parameters of the model are calculated as formula:

$$Z = -(A^T A)^{-1} A^T L. \tag{4}$$

Error of the model:

$$m_{MH} = \sqrt{\frac{[v^2]}{n-k}}. \tag{5}$$

2.2. Some of the subsidence models over time (Tran et al., 2017; Tran & Nguyen, 2010)

Exponential function: usually applied in analyzing the subsidence of civil works (high-rise buildings)

$$S = S_{TP}(1 - e^{-\alpha t}). \tag{6}$$

Polynomial function

$$S_t = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n, \tag{7}$$

where S_t – the subsidence of structure at time (t); t – the time that the subsidence S_t occurs; a_0, a_1, a_2, ...a_n – coefficients in polynomial equation; n – degree of polynomial equation.

Asaoka function

The generality formula of Asaoka function has form as follows:

$$S_{t_i} = \beta_0 + \beta_1 S_{t_{i-1}}, \tag{8}$$

where, S_{t_i}, $S_{t_{i-1}}$ – the subsidence at time (t_i) and (t_{i-1}); β_0, β_1 – coefficients of function.

Hyperbolic function

$$S_{t_i} = S_0 + \frac{t_i}{\alpha + \beta t_i}, \tag{9}$$

where, S_{t_i}, S_0 – the subsidence at time (t_i) and the initial time, respectively; α, β – coefficients of function.

Each subsidence function is suitable for each type of work or a part of the work depending on its structural characteristics. However, among the subsidence models mentioned, the polynomial function called the "almighty function" can substitute for any function. If it is not possible to determine the rule of subsidence of the building, it can choose the polynomial function as an approximation one. Therefore, the article chose the polynomial function as a model used in analyzing and predicting settlement of hydroelectric dams.

3. Kalman filter

Filter is an optimal recursive data processing algorithm for estimating the state vectors (state variables) of a dynamic system created by a data set containing random errors. There are many filter techniques, but the typical filter has been mentioned the most is Kalman filter (Gibbs, 2011; Ogundare, 2019; Kalman, 1960).

Dynamic filter model is an studied object for application in forecasting the structural settlement. A dynamic system is a cause-feedback system. Its memory stores the state of the system called a function over time, this function is used to predict the next state of the system in the future. Consider the dynamic system as a moving car, the unknown variables will constitute the elements of the state vector depending on time. These time-dependent vectors can forecast at any time through dynamic models or forecasting equations. The predictive value is updated with measurement values containing information about the components of the state vector. The Kalman filter can be considered as a form of least squares that allows parameters to vary over time.

Forecasting and filtering are closely related. The mathematical form of the filter consists of two independent models: a dynamic model (for forecasting) and a measurement values model (for updating forecast values).

3.1. The prediction model

Forecasting for the next cycle using the corrected state vector in the previous cycle (filter state vector).

The state predictive value at time (i) is determined by the formula:

$$\bar{X}_i = H_{tran_i} \hat{X}_{i-1} + B_i u_i + w_i, \tag{10}$$

where $\overline{X_i}$ – the predicted state vector at time t_i; $\hat{X}_{i-1}$ – the filtered state vector at time t_{i-1}; H_{tran_i} – the state transition matrix from t_{i-1} to t_i, or called the Jacobian matrix of the dynamic model; B_i – the control matrix for the effects of the input elements in the vector u_i; u_i – vector of control inputs of the dynamic system.

The dynamic system control factors in the vector u_i cannot be determined without knowing their conditions and effect rules. $B_i u_i$ represents additional information that is not related to the state itself, but it affects the system, therefore, it needs to be corrected in the forecast value. However, this component can be eliminated in a simple system on which no acted by external forces.

w_i – noise vector represents the dynamic model, is the random error of the model independing on the previous or next period, has an average value of 0, can set w = 0, the formula for calculating the prediction value is rewritten as follows:

$$\bar{X}_i = H_{tran_i} \hat{X}_{i-1} + B_i u_i. \tag{11}$$

The covariance matrix of the predicted state vector formed by the propagation law of the covariance function is given:

$$Q_{\bar{X}_i} = H_{tran_i} Q_{\hat{X}_{i-1}} H^T_{tran_i} + B_i Q_{u_i} B_i^T. \tag{12}$$

3.2. The filter model

Correcting in the predicted value according to 4 steps:

Establishing the model of measured values:

$$l_i = A_i \hat{X}_i + v_i, \tag{13}$$

where l_i – vector of measured values in the i^{th} period; A_i – coefficient matrix of measured values model; v_i – noise vector of measured values model.

Caculate the *Kalman gain* coefficient:

$$K_i = Q_{\bar{X}_i} A_i^T (A_i Q_{\bar{X}_i} A_i^T + Q_{l_i})^{-1}. \tag{14}$$

Update the predicted state vector to obtain the filter state vector:

$$\hat{X}_i = \bar{X}_i + K_i (l_i - A_i \bar{X}_i). \tag{15}$$

Update covariance matrix of the predicted state vector, obtain covariance matrix of the filter state vector:

$$Q_{\hat{X}_i} = (E - K_i A_i) Q_{\hat{X}_i}^T. \tag{16}$$

Calculate process of Kalman filter is illustrated in the diagram (Figure 1).

To predict subsidence of dam over time, it is necessary to eliminate subsidence caused by elevation of water level in the reservoir, method of determining the influence of elevation of water level on dam is presented in the next section.

4. Case study

4.1. Characteristics of the Hoa Binh hydroelectric dam

Hoa Binh hydroelectric plant has the dam that was built on the Da river, in Hoa Binh province in the north of Vietnam. It has twelve water discharging gates, eight generator units. Area of reservoir is 208 kilometer square with capacity of 9.45 billion cubic meters of water. This was the largest works in Vietnam and Southeast Asia from 1994 to 2012.

4.1.1. Characteristics of monitoring system for the Hoa Binh dam

Hoa Binh dam was monitored according to circle from 1987, includes four independent leveling lines established from fifty two monitoring points that are distributed on dam crest and body. The monitoring method is always geometric leveling. Although nonitored data from 1987 to 2020 was collected, selected data for analyzing and predicting experiment is subsidence and elevation of water level that were measured in the phase (2013–2015) when dam worked stably, at two points SM8 (on the dam body) and PVM8 (on the dam crest) (Figure 2). These are typical points and were monitored most sufficiently.

Figure 2. Image of hydroelectric dam in Hoa Binh

4.1.2. Monitoring results

From monitored data (Electricity of Vietnam, Hoa Binh Hydropower Company, 2020), a chart of relationship between elevation of water level and subsidence of Hoa Binh dam in over 200 periods was drawn as Figure 3. Smooth some small parts of chart, this dependence relationship is presented more clearly in Figure 4.

From mentioned charts, subsidence of Hoa Binh dam was affected by elevation of water level, when water rises, the displacement direction of dam is up, otherwise, water is lower, the displacement direction of dam is down.

Monitoring data of PVM8 and SM8 was used for experiment, shown in the Table 1.

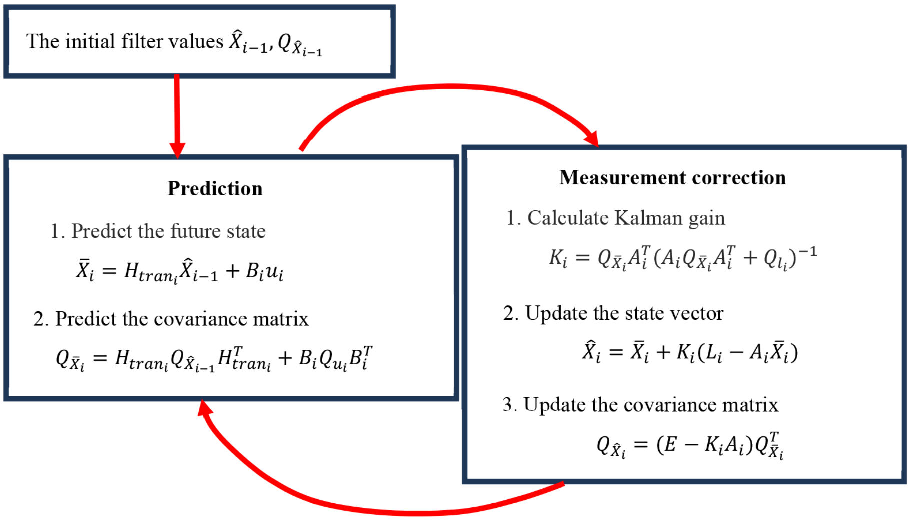


Figure 1. Diagram of calculation process of Kalman filter method (Gibbs, 2011; Welch & Bishop, 2006)

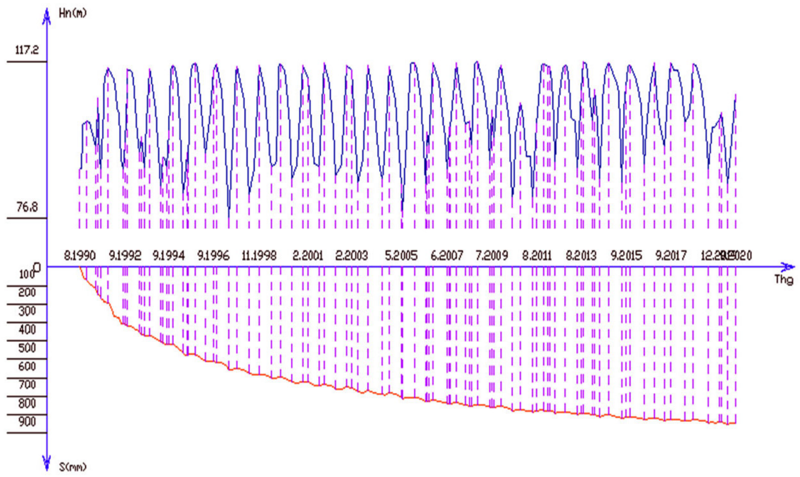
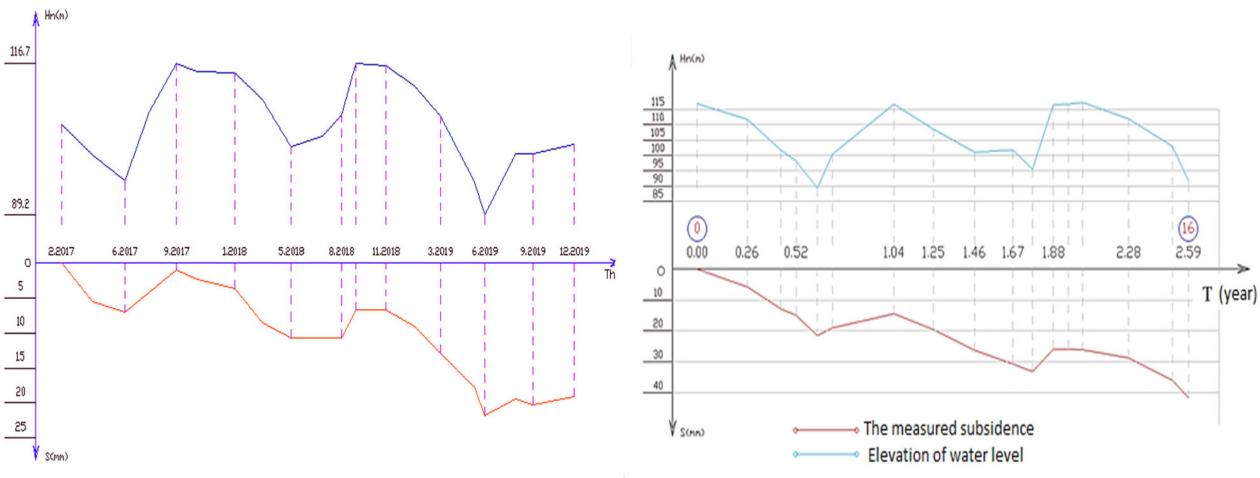


Figure 3. Relationship between elevation of water level and dam subsidence from 1990 to 2020



a) b)

Figure 4. Dependence relationship of subsidence on elevation of water level in some periods

Table 1. Results of subsidence monitoring at points PVM8 and SM8

Name of circles	Monitoring time	Subsidence (m)		Elevation of water level (m)
		PVM8	SM8	
159	29/1/2013	0.0000	0.0000	111.51
160	22/2/2013	–0.0041	–0.0012	102.71
161	2/5/2013	–0.0074	–0.0023	101.82
162	11/6/2013	–0.0119	–0.0038	89.62
163	15/8/2013	–0.0074	–0.0029	109.80
164	19/9/2013	–0.0035	–0.0036	117.20
165	12/11/2013	–0.0041	–0.0039	116.41
166	14/2/2014	–0.0140	–0.0063	101.86
167	28/3/2014	–0.0134	–0.0076	109.91
168	13/5/2014	–0.0184	–0.0090	101.85
169	25/6/2014	–0.0248	–0.0109	86.84
170	31/7/2014	–0.0210	–0.0108	104.09
171	15/8/2014	–0.0195	–0.0103	107.59
172	26/9/2014	–0.0160	–0.0105	115.40
173	10/11/2014	–0.0156	–0.0106	116.72

End of Table 1

Name of circles	Monitoring time	Subsidence (m)		Elevation of water level (m)
		PVM8	SM8	
174	2/2/2015	–0.0207	–0.0134	114.32
175	10/4/2015	–0.0231	–0.0159	111.60
176	11/5/2015	–0.0241	–0.0160	109.11
177	19/6/2015	–0.0356	–0.0197	85.83
178	5/8/2015	–0.0320	–0.0186	104.68
179	3/9/2015	–0.0288	–0.0183	110.82
180	6/10/2015	–0.0266	–0.0182	115.85
181	13/11/2015	–0.0242	–0.0163	116.38

4.2. Determining the influence of elevation of water level in the reservoir on the settlement of hydroelectric dam

4.2.1. Reasoning and algorithms

The subsidence of hydroelectric dam is calculated through polynomial function as form Eq. (7). At the initial time t_0 = 0, the structure has no subsidence (S_0 = 0), so a_0 = 0 (a_0 is coefficient in the polynomial Equation (7)). Therefore, the subsidence function over time is shown as follows:

$$S_t = a_1 t + a_2 t^2 + \ldots + a_n t^n. \quad (17)$$

The hydroelectric dam is affected not only by the subsidence over time, but also by the elevation of water level in the reservoir. Influence of the elevation of water level in the reservoir on the dam is determined as formula (US. Army Corps of Enginners, 2018):

$$S_H = u_0 + u_1 H + u_2 H^2 + \ldots + u_m H^m, \quad (18)$$

where u_0, u_1,......, u_m – coefficients in the polynomial function; H – the elevation of water level in the reservoir at time (t); m – degree of polynomial equation.

The measured subsidence is sum of the subsidence over time (S_t) and the subsidence caused by the elevation of water level in the reservoir S_{H_w}, it means:

$$S = S_t + S_H. \quad (19)$$

Due to the annually periodic change of the elevation of water level, it is easy to find out circles with the same elevation of water level. When the elevation of water level in the reservoir in different period is equal, influence of elevation of water level on subsidence of structure is similar. Based on this property, algorithms for determining the subsidence caused by the height of the water level is established as follows.

Assume that the water level in the reservoir in two monitoring periods i and j is approximately equal $(S_H^i \approx S_H^j)$. In monitoring periods i, j the measured settlement is calculated as the formulas:

$$S^i = S_t^i + S_H^i; \quad (20)$$

$$S^j = S_t^j + S_H^j. \quad (21)$$

Calculate the difference of subsidence between two monitoring periods, i.e. let Eq. (21) minus Eq. (20), given:

$$\Delta S^{ij} = (S^j - S^i) = S_t^j - S_t^i. \quad (22)$$

Thus, in Eq. (22), the difference of subsidence between the two monitoring periods is no longer affected by the elevation of the water level in the reservoir. Expanding the expression Eq. (22):

$$\Delta S^{ij} = a_1(t_j - t_i) + a_2(t_j^2 - t_i^2) + \ldots + a_n(t_j^n - t_i^n). \quad (23)$$

If ΔS^{ij} is considered as a measured value and the periods with the same water level are selected, a system of correction equations can be established as follows:

$$\begin{cases} V_{\Delta S_1} = a_1(t_j - t_i)_1 + a_2(t_j^2 - t_i^2)_1 + \ldots + a_n(t_j^n - t_i^n)_1 - \Delta S_1^{ij} \\ V_{\Delta S_2} = a_1(t_j - t_i)_2 + a_2(t_j^2 - t_i^2)_2 + \ldots + a_n(t_j^n - t_i^n)_2 - \Delta S_2^{ij} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ V_{\Delta S_k} = a_1(t_j - t_i)_k + a_2(t_j^2 - t_i^2)_k + \ldots + a_n(t_j^n - t_i^n)_k - \Delta S_k^{ij} \end{cases}. \quad (24)$$

Or written as the matrix: $V = AX + L$, where

$$A = \begin{bmatrix} (t_j - t_i)_1 & (t_j^2 - t_i^2)_1 & \ldots & (t_j^n - t_i^n)_1 \\ (t_j - t_i)_2 & (t_j^2 - t_i^2)_2 & & (t_j^n - t_i^n)_2 \\ & & & \\ (t_j - t_i)_k & (t_j^2 - t_i^2)_k & & (t_j^n - t_i^n)_k \end{bmatrix};$$

$$X = \left(a_1 \;\; a_2 \;\; \ldots \;\; a_n\right)^T; \quad (25)$$

$$L = \left(-\Delta S_1^{ij} \;\; -\Delta S_2^{ij} \;\; \ldots \;\; -\Delta S_k^{ij}\right)^T.$$

If $k > n$, then the system Eq. (24) is solved by the principle of least squares with the condition $\left[V_{\Delta S}^2\right] = \min$. The coefficients ($a$) in the variable vector are determined by the formula:

$$X = -(A^T A)^{-1} A^T L. \quad (26)$$

After the preliminarily determination of the coefficients (a), calculate the subsidence S_t as (17). Then, calculate the influence of water level on subsidence of dam for all cycles as follows:

$$S_H = S_{do} - S_t. \quad (27)$$

Based on Eqs (18) and (27), apply the principle of least squares for determining the coefficients (u) of the function of settlement caused by the elevation of the water level as the following process:

Establish a system of correction equations:

$$V_{S_H} = u_0 + u_1 H + u_2 H^2 + \ldots + u_m H^m - S_H \quad (28)$$

or matrix form:

$$V' = A'U + L', \quad (29)$$

where:

$$A' = \begin{pmatrix} 1 & H_1 & \ldots & H_1^m \\ 1 & H_2 & \ldots & H_2^m \\ \ldots & \ldots & \ldots & \ldots \\ 1 & H_t & \ldots & H_t^m \end{pmatrix};$$

$$U = \left(u_0 \;\; u_1 \;\; \ldots \;\; u_m\right)^T; \quad (30)$$

$$L' = \left(-S_{H_1} \;\; -S_{H_2} \;\; \ldots \;\; -S_{H_m}\right)^T.$$

Establish the standard equation system

$$(A')^T A'U + (A')^T L' = 0. \quad (31)$$

Calculate the variable vector U:

$$U = -((A')^T A')^{-1}(A')^T L. \quad (32)$$

The error of model

$$\mu = \sqrt{\frac{[vv]}{N - (m+1)}}. \quad (33)$$

4.2.2. Calculation process

Based on the reasoning of the problem, the calculation process was built including 2 steps as follows:

Stage 1. Approximately determine coefficients u in the settlement function according to the elevation of water level in the reservoir with the condition $\left[V_{\Delta S}^2\right] = \min$. Use cycles with the nearly equal water level.

Step 1: Determine the coefficient (a) of the settlement function over time, calculate S_t for periods with the approximately equal water level.

Step 2: Calculate $S_H = S_{do} - S_t$ for all cycles.

Step 3: Determine the coefficient (u) of the settlement function of elevation of the water level.

Stage 2. Calculate the coefficient (u) with the condition $\left[V_S^2\right] = \min$. Use all the cycles in the data set.

Step 1: From S_H calculated on the stage 1, determine the values S_t for all cycles and find the coefficient (a) of the settlement function over time.

Step 2: Calculate S_H, then find the coefficient (u) of the settlement function of the elevation of water level.

After determining the influence of water level on settlement, calculate $S_t = S_{do} - S_H$, continue the process of finding a, u. Repeat the calculation until a, u converge.

4.2.3. Selection of the degree of polynomial

Replace sequently the degree of polynomial from the smallest one (n = 1, m = 1) for settlement functions over time and settlement function of the elevation of the water level.

For each polynomial assigned the degree, implement the calculation process in 2 stages as mentioned (Figure 5) to determine the coefficients a, u and the error of model.

The degree of polynomial is chosen properly when the polynomial has the error of model (formula (33)) that is equivalent to the measurement error (Tran & Nguyen, 2010).

4.3. Subsidence prediction

Process of subsidence prediction for hydroelectric dam as follows.

4.3.1. Establishment of prediction model

Composition of subsidence over time in general form

$$S_{t_i} = a_1 t_i. \tag{34}$$

At the time (t_i), velocity of subsidence over time is

$$v_{t_i} = \frac{\partial S_{t_i}}{\partial t} = a_1. \tag{35}$$

Therefore, difference of subsidence between the time (i) and (i – 1):

$$S_{t_i} - S_{t_{i-1}} = a_1(t_i - t_{i-1}). \tag{36}$$

Subsidence over time at time (i) can be rewrited as follows:

$$S_{t_i} = S_{t_{i-1}} + a_{1(i-1)}(t_i - t_{i-1}). \tag{37}$$

Composition of subsidence caused by elevation of water level in general form

$$S_{H_i} = u_0 + u_1 H_i. \tag{38}$$

From the time (i) to (i – 1), elevation of water level in the reservoir is considered to change linearly, so elevation of water level at the time (t) between two monitoring circles is calculated:

$$H_t = \frac{(H_i - H_{i-1})}{(t_i - t_{i-1})}(t - t_{i-1}) + H_{i-1}. \tag{39}$$

From Eqs (38) and (39), velocity of subsidence caused by elevation of water level is detemined as follows:

$$v_{S_{Hi}} = \frac{\partial S_{H_i}}{\partial t} = u_1 \frac{(H_i - H_{i-1})}{(t_i - t_{i-1})} = u_1 \frac{\Delta H_{i,i-1}}{\Delta t_{i,i-1}}. \tag{40}$$

Similar to formular (36), calculate difference of subsidence caused by elevation of water level between the time (i) and (i – 1), thus subsidence caused by elevation of water level at time (i) can be rewrited as follows:

$$S_{H_i} - S_{H_{i-1}} = u_1(H_i - H_{i-1}). \tag{41}$$

Velocity of subsidence caused by elevation of water level at time (t):

$$v_{S_H} = u_1 \left(\frac{\Delta H_{i,\,i-1}}{\Delta t_{i,\,i-1}} - \frac{\Delta H_{i-1,\,i-2}}{\Delta t_{i-1,\,i-2}} \right). \tag{42}$$

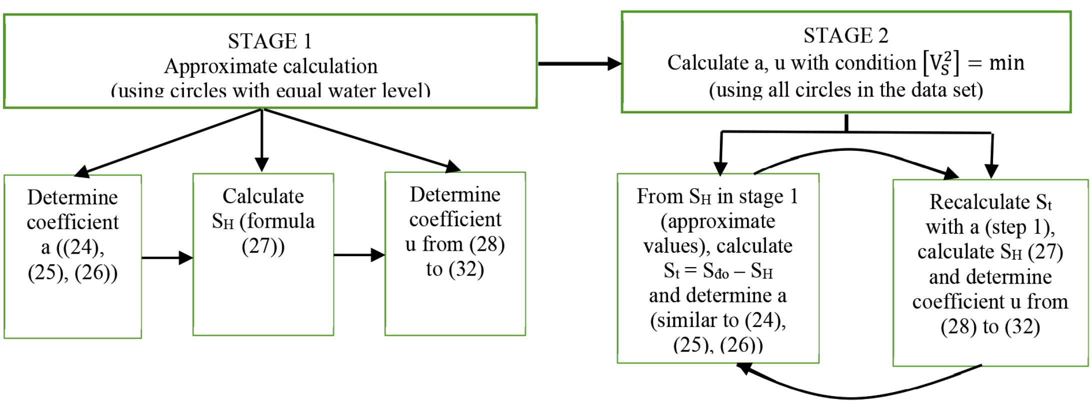


Figure 5. Calculation process for determining impact of water level on subsidence

Symbol the filtered state vector at time (i – 1) as $\hat{X}_{i-1}=\left[\hat{s}_{i-1}\ \ \hat{a}_{i-1}\right]^T$.

Symbol the predicted state vector at time (i) as $\bar{X}_i=\left[\bar{S}_i\ \ \bar{a}_i\right]^T$.

Kalman filter is built based on subsidence and settlement velocity over time. From formulas (37), (41), determining subsidence in the i[th] monitoring circle as given:

$$\bar{S}_i=\hat{S}_{i-1}+\hat{a}_{1(i-1)}(t_i-t_{i-1})+u_1(H_i-H_{i-1}). \tag{43}$$

Velocity of subsidence at time (t) between two monitoring circles (i) and (i – 1) is determined from formulas (35), (42):

$$\bar{a}_i=\hat{a}_{i-1}+u_1\left(\frac{\Delta H_{i,\,i-1}}{\Delta t_{i,\,i-1}}-\frac{\Delta H_{i-1,\,i-2}}{\Delta t_{i-1,\,i-2}}\right). \tag{44}$$

Due to no determining the impact conditions of u_1 on dynamic system, so subsidence caused by elevation of water level is eliminated out of the measured subsidence values (remove B_iu_i in formula (11)), then the predicted values are calculated as follows:

$$\bar{X}_i=H_i\hat{X}_{i-1}. \tag{45}$$

The state transition matrix H_{tran} is determined as given

$$H=\begin{pmatrix}\dfrac{\partial\bar{S}_i}{\partial\hat{S}_{i-1}} & \dfrac{\partial\bar{S}_i}{\partial\hat{a}_{i-1}}\\ \dfrac{\partial\bar{a}_i}{\partial\hat{S}_{i-1}} & \dfrac{\partial\bar{a}_i}{\partial\hat{a}_{i-1}}\end{pmatrix}=\begin{pmatrix}1 & \Delta t_{i,i-1}\\ 0 & 1\end{pmatrix}. \tag{46}$$

Despite no determing the impact conditions, when assessing influence of elevation of water level on dam subsidence, error of u_1 was found, so it is necessary to correct this error into covariance matrix of the predicted state vector according to formula (12).

In formula (12), The control matrix for the effects of the input elements B in the vector u has the form:

$$B=\begin{pmatrix}\dfrac{\partial\bar{S}_i}{\partial u_1}\\ \dfrac{\partial\bar{a}_i}{\partial u_1}\end{pmatrix}=\begin{pmatrix}\Delta H_{i,i-1}\\ \dfrac{\Delta H_{i,i-1}}{\Delta t_{i,i-1}}-\dfrac{\Delta H_{i-1,\,i-2}}{\Delta t_{i-1,\,i-2}}\end{pmatrix}. \tag{47}$$

4.3.2. Establishment of the measured values model

Equation of measured values has the form as follows:

$$l_i=S_i. \tag{48}$$

Experiment of forecasting subsidence of two points PVM8 and SM8.

Firsly, calculate subsidence over time (eliminate subsidence caused by elevation of water level).

Secondly, implement the process of Kalman filter according to loop circle in 13 monitoring periods.

Finally, predict subsidence of the last eight monitoring periods.

5. Results analysis and discussions

5.1. Subsidence caused by elevation of water level

Actually, subsidence caused by elevation of water level is noise part which is eliminated out of the measured subsidence before predicting subsidence of dam.

At point PVM8

Subsidence function:

$$S_H=-0.047032+0.0004218H. \tag{49}$$

Calculated result is shown in Table 2.

Table 2. Subsidence caused by elevation of water level at PVM8

Name of circles	Monitoring time	Elevation of water level (m)	Subsidence S_H (m)
159	29/1/2013	111.51	0.0000
160	22/2/2013	102.71	–0.0037
161	2/5/2013	101.82	–0.0041
162	11/6/2013	89.62	–0.0092
163	15/8/2013	109.80	–0.0007
164	19/9/2013	117.20	0.0024
165	12/11/2013	116.41	0.0021
166	14/2/2014	101.86	–0.0041
167	28/3/2014	109.91	–0.0007
168	13/5/2014	101.85	–0.0041
169	25/6/2014	86.84	–0.0104
170	31/7/2014	104.09	–0.0031
171	15/8/2014	107.59	–0.0017
172	26/9/2014	115.40	0.0016
173	10/11/2014	116.72	0.0022
174	2/2/2015	114.32	0.0012
175	10/4/2015	111.60	0.0001
176	11/5/2015	109.11	–0.0010
177	19/6/2015	85.83	–0.0108
178	5/8/2015	104.68	–0.0029
179	3/9/2015	110.82	–0.0003
180	6/10/2015	115.85	0.0018
181	13/11/2015	116.38	0.0021

Results are presented on diagram as follows (Figure 6).

At point SM8

Subsidence function:

$$S_H=-0.008495+0.0000762H. \tag{50}$$

Calculated result is shown in Table 3.

Results are presented on diagram as follows (Figure 7).

Comment: Influence of elevation of water level on two different points of dam is not similar although elevation of water level is the same.

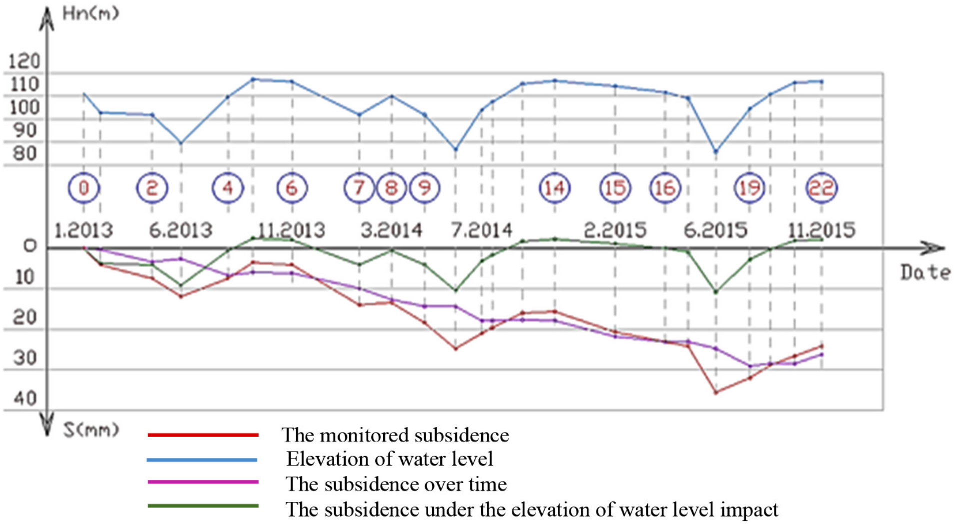


Figure 6. Subsidence caused by elevation of water level at the monitoring point PVM8

Table 3. Subsidence caused by elevation of water level at SM8

Name of circles	Monitoring time	Elevation of water level (m)	Subsidence S_H (m)
159	29/1/2013	111.51	0.0000
160	22/2/2013	102.71	–0.0007
161	2/5/2013	101.82	–0.0007
162	11/6/2013	89.62	–0.0017
163	15/8/2013	109.80	–0.0001
164	19/9/2013	117.20	0.0004
165	12/11/2013	116.41	0.0004
166	14/2/2014	101.86	–0.0007
167	28/3/2014	109.91	–0.0001
168	13/5/2014	101.85	–0.0007
169	25/6/2014	86.84	–0.0019
170	31/7/2014	104.09	–0.0006

End of Table 3

Name of circles	Monitoring time	Elevation of water level (m)	Subsidence S_H (m)
171	15/8/2014	107.59	–0.0003
172	26/9/2014	115.40	0.0003
173	10/11/2014	116.72	0.0004
174	2/2/2015	114.32	0.0002
175	10/4/2015	111.60	0.0000
176	11/5/2015	109.11	–0.0002
177	19/6/2015	85.83	–0.0019
178	5/8/2015	104.68	–0.0005
179	3/9/2015	110.82	–0.0001
180	6/10/2015	115.85	0.0003
181	13/11/2015	116.38	0.0004

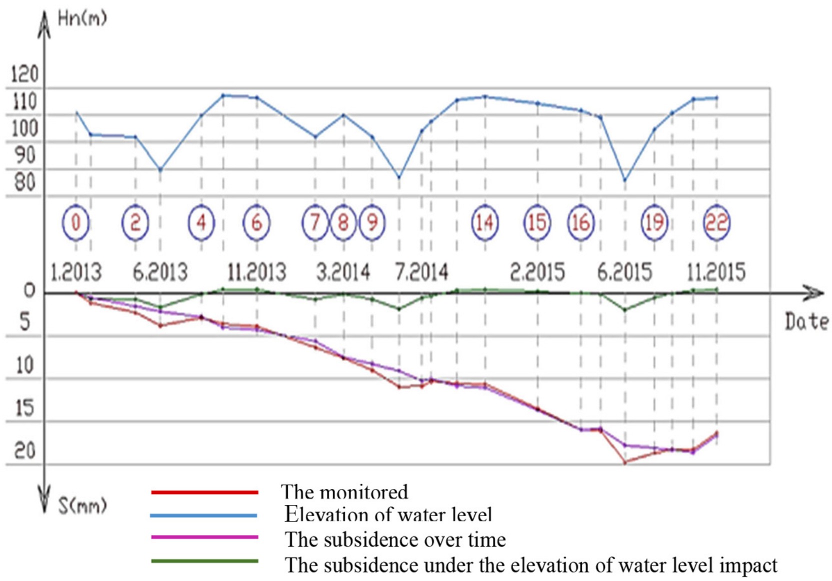


Figure 7. Subsidence caused by elevation of water level at the monitoring point SM8

5.2. Results of subsidence prediction

At point PVM8

Subsidence over time is shown in the Table 4.

Table 4. Subsidence over time of point PVM8

No	S_{do} (m)	S_H (m)	S_T (m)
159	0.0000	0.0000	0.0000
160	–0.0041	–0.0037	–0.0004
161	–0.0074	–0.0041	–0.0033
162	–0.0119	–0.0092	–0.0027
163	–0.0074	–0.0007	–0.0067
164	–0.0035	0.0024	–0.0059
165	–0.0041	0.0021	–0.0062
166	–0.0140	–0.0041	–0.0099
167	–0.0134	–0.0007	–0.0127
168	–0.0184	–0.0041	–0.0143
169	–0.0248	–0.0104	–0.0144
170	–0.0210	–0.0031	–0.0179

End of Table 4

No	S_{do} (m)	S_H (m)	S_T (m)
171	–0.0195	–0.0017	–0.0178
172	–0.0160	0.0016	–0.0176
173	–0.0156	0.0022	–0.0178
174	–0.0207	0.0012	–0.0219
175	–0.0231	0.0001	–0.0232
176	–0.0241	–0.0010	–0.0231
177	–0.0356	–0.0108	–0.0248
178	–0.0320	–0.0029	–0.0291
179	–0.0288	–0.0003	–0.0285
180	–0.0266	0.0018	–0.0284
181	–0.0242	0.0021	–0.0263

The initial filter values

$$Q_{\hat{X}} = \begin{bmatrix} 0.012 & 0.192 \\ 0.192 & 2.997 \end{bmatrix};$$

$$\hat{X}_1 = \begin{bmatrix} -0.0004 & -0.0061 \end{bmatrix}^T.$$

Results are shown in Table 5 and Table 6.

Table 5. Results of calculation process using Kalman filter

Name of circles	$Q_{\bar{X}}$	$\bar{X}$		$\hat{X}$		$Q_{\hat{X}}$
		$\bar{S}$	$\bar{a}_1$	$\hat{S}$	$\hat{a}_1$	
2	$\begin{bmatrix} 0.202 & 0.776 \\ 0.776 & 3.142 \end{bmatrix}$	–0.0016	–0.0061	–0.0032	–0.0124	$\begin{bmatrix} 0.012 & 0.045 \\ 0.045 & 0.325 \end{bmatrix}$
3	$\begin{bmatrix} 0.026 & 0.091 \\ 0.091 & 0{,}422 \end{bmatrix}$	–0.0045	–0.0124	–0.0033	–0.0080	$\begin{bmatrix} 0.008 & 0.029 \\ 0.029 & 0.209 \end{bmatrix}$
4	$\begin{bmatrix} 0.029 & 0.104 \\ 0.104 & 0.630 \end{bmatrix}$	–0.0047	–0.0080	–0.0061	–0.0130	$\begin{bmatrix} 0.009 & 0.031 \\ 0.031 & 0.368 \end{bmatrix}$
5	$\begin{bmatrix} 0.018 & 0.064 \\ 0.064 & 0.378 \end{bmatrix}$	–0.0073	–0.0130	–0.0065	–0.0101	$\begin{bmatrix} 0.007 & 0.026 \\ 0.026 & 0.245 \end{bmatrix}$
6	$\begin{bmatrix} 0.020 & 0.062 \\ 0.062 & 0.303 \end{bmatrix}$	–0.0080	–0.0101	–0.0068	–0.0067	$\begin{bmatrix} 0.008 & 0.024 \\ 0.024 & 0.183 \end{bmatrix}$
7	$\begin{bmatrix} 0.033 & 0.077 \\ 0.077 & 0.205 \end{bmatrix}$	–0.0085	–0.0067	–0.0096	–0.0090	$\begin{bmatrix} 0.009 & 0.021 \\ 0.021 & 0.077 \end{bmatrix}$
8	$\begin{bmatrix} 0.016 & 0.038 \\ 0.038 & 0.201 \end{bmatrix}$	–0.0106	–0.0090	–0.0118	–0.0118	$\begin{bmatrix} 0.007 & 0.017 \\ 0.017 & 0.149 \end{bmatrix}$
9	$\begin{bmatrix} 0.014 & 0.044 \\ 0.044 & 0.289 \end{bmatrix}$	–0.0133	–0.0118	–0.0138	–0.0135	$\begin{bmatrix} 0.007 & 0.021 \\ 0.021 & 0.215 \end{bmatrix}$
10	$\begin{bmatrix} 0.016 & 0.054 \\ 0.054 & 0.249 \end{bmatrix}$	–0.0154	–0.0135	–0.0148	–0.0116	$\begin{bmatrix} 0.007 & 0.023 \\ 0.023 & 0.148 \end{bmatrix}$
11	$\begin{bmatrix} 0.016 & 0.081 \\ 0.081 & 0.897 \end{bmatrix}$	–0.0160	–0.0116	–0.0170	–0.0170	$\begin{bmatrix} 0.007 & 0.036 \\ 0.036 & 0.662 \end{bmatrix}$
12	$\begin{bmatrix} 0.011 & 0.059 \\ 0.059 & 0.721 \end{bmatrix}$	–0.0177	–0.0170	–0.0178	–0.0174	$\begin{bmatrix} 0.006 & 0.032 \\ 0.032 & 0.568 \end{bmatrix}$
13	$\begin{bmatrix} 0.021 & 0.095 \\ 0.095 & 0.571 \end{bmatrix}$	–0.0197	–0.0174	–0.0184	–0.0113	$\begin{bmatrix} 0.008 & 0.035 \\ 0.035 & 0.299 \end{bmatrix}$
14	$\begin{bmatrix} 0.021 & 0.071 \\ 0.071 & 0.327 \end{bmatrix}$	–0.0198	–0.0113	–0.0185	–0.0070	$\begin{bmatrix} 0.008 & 0.026 \\ 0.026 & 0.174 \end{bmatrix}$

Table 6. Result of subsidence prediction for last 8 cycles

No	Monitoring time		Elevation of water level (m)	Predicted subsidence (m)	Accuracy of prediction	Measured subsidence values (m)	Difference (m)
	Date-month-year	Compared to circle 14					
15	2-2-2015	2.74	114.32	–0.0190	0.0017	–0.0207	0.0017
16	10 4-2015	5.00	111.60	–0.0214	0.0034	–0.0231	0.0017
17	11-5-2015	6.03	109.11	–0.0231	0.0055	–0.0241	0.0010
18	19-6-2015	7.30	85.83	–0.0336	0.0081	–0.0356	0.0020
19	5-8-2015	8.84	104.68	–0.0266	0.0126	–0.0320	0.0054
20	3-9-2015	9.77	110.82	–0.0245	0.0189	–0.0288	0.0043
21	6-10-2015	10.87	115.85	–0.0231	0.0265	–0.0266	0.0035
22	13-11-2015	12.10	116.38	–0.0236	0.0353	–0.0242	0.0006

Model of Subsidence over time

$S = -0.0070t.$

At point SM8

Similarly, predict subsidence of point SM8, results are shown in Table 7

Subsidence model

$S = -0.0055t.$

The obtained results is in good agreement with the monitoring results (Figures 8, 9). Therefore, the obtained results are considered accurate and reliable. Based on the results, it can be stated that the Kalman filter is effective for predicting subsidence of hydroelectric dam.

Table 7. Result of subsidence prediction in 2015

No	Monitoring time	Predicted subsiden-ce (m)	Accuracy of predic-tion (m)	Measured subsidence values (m)	Diffe-rence (m)
174	2/2/2015	–0.0123	0.0007	–0.0134	0.0011
175	10/4/2015	–0.0136	0.0014	–0.0159	0.0023
176	11/5/2015	–0.0142	0.0022	–0.0160	0.0018
177	19/6/2015	–0.0166	0.0033	–0.0197	0.0031
178	5/8/ 2015	–0.0159	0.0051	–0.0186	0.0027
179	3/9/ 2015	–0.0158	0.0076	–0.0183	0.0025
180	6/10/2015	–0.0159	0.0107	–0.0182	0.0023
181	13/11/2015	–0.0165	0.0143	–0.0163	–0.0002

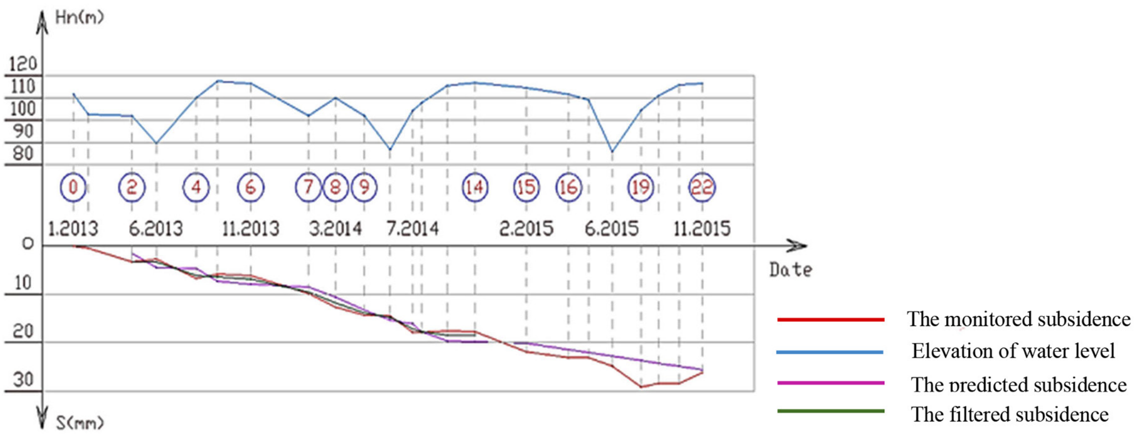


Figure 8. Predicted subsidence at the monitoring point PVM8

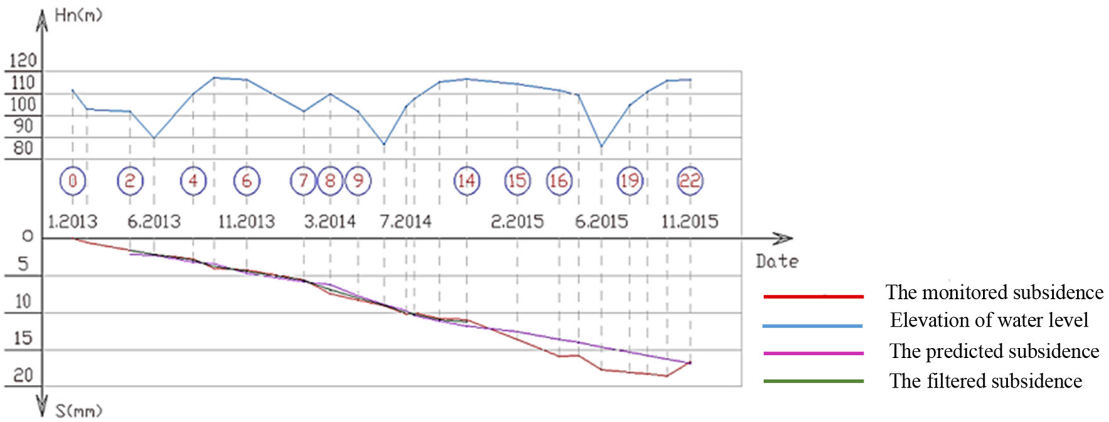


Figure 9. Predicted subsidence at the monitoring point SM8

6. Conclusions

In the present study, a subsidence analysis was conducted for the Hoa Binh hydroelectric dams using the Kalman filter method. An external factor – elevation of water level in the reservoir was taken into account to predict the subsidence of the given dam. The following conclusions can be drawn:

Kalman filter is one of the feasible methods that have been used in analysing and predicting subsidence. In the predicted model, there are two major components: calculated subsidence value and impact of external factors. Therefore, the method can filter the subsidence caused by influencing factors and eliminate them out of the monitoring outcomes.

Results clearly indicate the impact of that elevation of water level in the reservoir subsidence at Hoa Binh dam. When water rises, the displacement direction of dam is up, otherwise, water is lower, the displacement direction of dam is down. Elevation of water level has a significant role in production of hydroelectric plant. Predicted subsidence would assist management to adjust water level suitably to maintain safety of dam and continuous production, simultaneously.

The predicted subsidence has a slight difference from the measured values. Kalman filter method can predict subsidence properly with high accuracy and reliability

Process of analysis and prediction in this paper can be used as reference for others hydroelectric dams in the similar conditions in Vietnam and over the world.

Hydroelectric dams should be monitored regularly and the monitoring outcomes should be analyzed and verified to ensure safety of dams

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