



INVESTMENT PROJECTS EVALUATION BY SIMULATION AND MULTIPLE CRITERIA DECISION AIDING PROCEDURE

Maciej Nowak

The Karol Adamiecki University of Economics, ul. 1 Maja 50, 40-287 Katowice, Poland.

E-mail: nomac@ae.katowice.pl

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Abstract. The paper considers an investment projects selection problem. The evaluation of each project is usually a multidimensional problem. On the one hand, financial analysis is very important, on the other, however, technical, social, and ecological factors are taken into account too. While financial criteria are usually of quantitative nature, others are often based on qualitative judgments. As the analysis of each project is based on uncertain assumptions, so the problem can be considered as a discrete stochastic multiple criteria decision-making problem. In this paper simulation, stochastic dominance rules and multiple criteria decision aiding procedure PROMETHEE II are employed for solving such a problem. While simulation technique is used for obtaining financial evaluations of projects, experts' judgments are taken into account in order to evaluate project with respect to other criteria. Thus, quantitative and qualitative factors are considered in this approach.

Keywords: project selection problem, multicriteria analysis, uncertainty modelling, simulation.

1. Introduction

Investment activity plays an extremely important role in every economic organization. Selection of a new project or a group of projects constitutes one of the main management functions required to ensure business survival. Various elements are usually taken into account when an investment project is analysed. Technical factors, environmental effects, social issues and financial profitability are of significant importance. Thus, project selection is a typical example of a multiple criteria decision-making problem. It is proper to add that evaluation criteria can be of different kinds. While financial analysis is usually conducted by quantitative measures like Net Present Value, Internal Rate of Return, Profitability Index or others, evaluation with respect to technical, environmental and social criteria is often of qualitative nature.

Project evaluation involves prediction of future outcomes. In real world, however, all predictions are not known with certainty. The fact of matter is that modern businesses face a more severe and challenging environment than ever before. The increasing volatility in interest and exchange rates, lifting trade barriers and development of new technologies in electronics and biotechnology result in a high level of uncertainty in managerial decision-making. Thus, a demand for new decision aiding techniques that can be employed in project evaluation and selection problems arises.

Numerous procedures have been proposed in recent years for evaluating engineering, procurement and construction projects based on an established set of objectives. Analytical Hierarchy Process (AHP), proposed by Saaty (1980), is one of the most widely employed techniques. This method is used, for example, by Ferrari (2003), and Kearns (2004). The main idea of AHP is to exploit the results of the decision-maker's subjective evaluations formulated for each pair of projects and for each criterion.

Techniques based on the utility function concept compose another group of methods employed in project selection problems. Such an approach is employed by Moselhi and Deb (1993), Graves and Rinqest (1996), Wong et al (2000). They propose techniques for estimating single-criteria utility functions and aggregating them into a multiattribute utility function.

Various methods based on the outranking relation concept are also proposed. Martel and D'Avignon (1982) establish a confidence index by using probabilities that one project is as good as another. Pin-Yu et al (1996), Costa et al (2003), Mavrotas et al (2003) use ELECTRE methods, while Al-Rashdan et al (1999), and Goumas et al (1999) employ PROMETHEE II technique in project evaluation problems.

Goal programming approach is also widely used. This concept is utilised, for example, by Santhanam and Kyprasis (1995), de Oliveira et al (2003), Lee and Kim (2000).

In this paper a decision aiding procedure for projects evaluation problem is proposed. This technique can successfully be employed for analysing manufacturing and construction projects, in which timetables are more or less predictable based on past experience. R&D projects in which such schedules cannot be precisely predicted may need a different technique.

Four main concepts are employed in this technique: simulation, experts judgements analysis, stochastic dominance and multiple criteria decision aiding procedure PROMETHEE II. While simulation is used for constructing the knowledge base for financial evaluation of each project, experts opinions are analysed when qualitative factors are taken into account. As a result, evaluations of each project with respect to each criterion are obtained. These evaluations take form of probability distributions. In the procedure proposed here stochastic dominance (SD) rules are used for comparing these distributions. Thus we can avoid estimating decision-maker's utility function, which is usually done in classical approach based on multiattribute utility function. PROMETHEE II technique is employed for generating final ranking of considered projects.

The paper is organised as follows. Section 2 gives a notation for all terms used in the manuscript. In section 3 project evaluation problem is formulated as a multiple criteria decision-making problem. Section 4 presents stochastic dominance rules. The multiple criteria decision-making procedure for project selection problem is presented in section 5. Section 6 gives an example. The last section is conclusion.

2. Notation used in the manuscript

The notation used in the manuscript:

t – period number,

CF_0 – amount of initial investment,

CF_t – cash flow in period t ,

r_t – discount rate in period t ,

T – the number of considered periods,

A – the set of alternatives (considered projects),

m – the number of alternatives,

a_i – alternative no i ,

X – the set of attributes,

n – the number of attributes,

X_k – attribute no k ,

E – the set of evaluations of alternatives with respect to attributes,

X_k^i – the evaluation of alternative a_i with respect to attribute X_k ,

$F_k^i(x)$ – right-continuous cumulative distribution function representing evaluations of a_i over attribute X_k ,

μ_k^i – average performance of the alternative a_i on the attribute X_k ,

$p_k(\mu_k^i)$ – preference threshold for attribute X_k ,

$v_k(\mu_k^i)$ – veto threshold for attribute X_k ,

R – the set of real numbers.

3. Project selection as a multiple criteria decision-making problem

Each serious investment decision is usually preceded by a detailed analysis aimed at investigating project effects. In fact, the investment decision is conditioned by multiple factors of various kinds.

Let us start with financial part of the analysis. Although various measures are proposed, project net present value (NPV) is generally considered as one of the most important. It is calculated as a difference between the sum of discounted values of future cash flows and the initial investment

$$NPV = \sum_{t=1}^n \frac{CF_t}{(1+r)^t} - CF_0.$$

Profitability index (PI) is a measure calculated as a ratio between discounted cash flows and initial investment

$$PI = \frac{\sum_{t=1}^n \frac{CF_t}{(1+r)^t}}{CF_0}.$$

Internal rate of return (IRR) is a discount rate that makes discounted cash flows equal to initial investment or, in other words, NPV equals zero. Thus, IRR can be calculated by using the formula

$$\sum_{t=1}^n \frac{CF_t}{(1+IRR)^t} - CF_0 = 0.$$

A large number of other measures can be used in various situations (Remer, Nieto, 1995a, 1995b). The calculations of all these measures are based on predicted values of future outcomes, and, as a result, their values cannot be estimated with certainty.

Although financial estimation is very important, other issues, including technical, social, environmental, and others are also considered when an investment project is analysed. Technical evaluation, for example, includes such issues like level of technical novelty, compatibility with other technologies used by the organisation, reliability and technical service, work safety and others. It is clear that these criteria are often of qualitative kind. The similar situation arises when social and environmental effects are considered. Although quantitative characteristics can sometimes be employed, in many cases the evaluations are of qualitative nature.

As various issues are taken into account, the investor faces a problem of a multidimensional comparison of investment alternatives. Thus our problem can be considered as a multiple criteria decision-making problem.

We assume that only one alternative can be selected. Each project is evaluated with respect to n attributes. In our problem some of them are of quantitative nature, while others are qualitative ones. While quantitative attributes are measured on cardinal scale, the qualitative ones are unquantifiable in this sense. The decision-maker

cannot evaluate the monetary values of alternatives with respect to such attributes. We assume, however, that the decision-maker is able to define potential outcomes and to rank these outcomes according to his/her preferences.

Our problem can be represented as *Alternatives, Attributes, Evaluations* (A, X, E) model. We consider:

1. A finite set of alternatives (investment projects):

$$A = \{ a_1, a_2, \dots, a_m \}$$

2. A finite set of attributes:

$$X = \{ X_1, X_2, \dots, X_n \}$$

3. A set of evaluations of alternatives with respect to attributes:

$$E = \begin{bmatrix} X_1^1 & \dots & X_j^1 & \dots & X_n^1 \\ \vdots & \dots & \vdots & \dots & \vdots \\ X_1^i & \dots & X_j^i & \dots & X_n^i \\ \vdots & \dots & \vdots & \dots & \vdots \\ X_1^m & \dots & X_j^m & \dots & X_n^m \end{bmatrix}.$$

There are two sources of data used in the procedure presented below: simulation and experts' valuations. A series of simulation experiments is conducted for each project. Thus, a sequence of observations for each alternative with respect to each attribute is obtained. These observations are used for constructing distributional evaluations.

At the same time several experts are asked to evaluate projects with respect to qualitative attributes. As a result, series of evaluations are obtained for each project with respect to each qualitative attribute. Again, these data are exploited for generating distributional evaluations.

4. Stochastic dominance rules

Various approaches are proposed for solving stochastic multiple criteria decision-making problem. Keeney and Raiffa (1976) proposed multiattribute utility function concept. They showed that if the additive utility independence condition is fulfilled, then multiattribute comparison can be decomposed to one-attribute comparisons. In such case the problem is solved by estimating one-attribute utility function and calculating multiattribute utility, which is a weighted sum of one-attribute utilities. Unfortunately, estimation of one-attribute utility function is an uneasy and time consuming task. By using stochastic dominance concept we can avoid estimating utility functions. If certain assumptions on the type of the decision-maker's utility functions are fulfilled, then stochastic dominance rules are equivalent to expected utility rules.

Let us assume that attributes are defined in such a way, that a larger value is preferred to a smaller one. Let $F_k^i(x)$ and $F_k^j(x)$ be right-continuous cumulative distribution functions representing evaluations of a_i and a_j respectively over attribute X_k :

$$F_k^i(x) = \Pr(X_k^i \leq x) \\ F_k^j(x) = \Pr(X_k^j \leq x).$$

Definitions of the first and second degree stochastic dominance relations are as follows:

Definition 1 – First Degree Stochastic Dominance:

X_k^i FSD X_k^j if and only if

$$F_k^i(x) \neq F_k^j(x) \text{ and } H_1(x) = F_k^i(x) - F_k^j(x) \leq 0 \text{ for } x \in R$$

Definition 2 – Second Degree Stochastic Dominance:

X_k^i SSD X_k^j if and only if

$$F_k^i(x) \neq F_k^j(x) \text{ and } H_2(x) = \int_{-\infty}^x H_1(y) dy \leq 0 \text{ for } x \in R.$$

Hadar and Russel (1969) showed that the FSD rule is equivalent to the expected utility rule for all decision-makers preferring larger outcomes, while the SSD rule is equivalent to the expected utility rule for risk-averse decision-makers preferring larger outcomes.

Rules defined above apply to outcomes measured on cardinal scales, such as income, wealth, rates of return and so on, but fail to provide ranking of preferences among variables of ordinal nature. Rules that can be applied in such situations have been proposed by Spector et al (1996). They distinguish two separate ordinal measurements:

1. The alternative outcomes can only be ranked in order of preference.
2. In addition to ranking, it is also possible to rank the differences between alternative outcomes.

Let us assume that the random variable X_k^i is defined by $(e_{k1}, \dots, e_{kz}, p_{k1}^i, \dots, p_{kz}^i)$, where e_{k1}, \dots, e_{kz} are z real numbers, such that $e_l < e_{l+1}$ for all $l = 1, \dots, z-1$, and $p_{k1}^i, \dots, p_{kz}^i$ are the probability measures. The variable X_k^j is defined similarly with $p_{k1}^j, \dots, p_{kz}^j$ replacing $p_{k1}^i, \dots, p_{kz}^i$.

If the outcomes can be ranked in order of preferences, i.e. the decision-maker prefers e_{l+1} over e_l for all $l = 1, \dots, z-1$, then Ordinal First Degree Stochastic Dominance (OFSD) rule can be used:

Definition 3 – Ordinal First Degree Stochastic Dominance:

X_k^i OFSD X_k^j if and only if

$$\sum_{l=1}^s p_{kl}^i \leq \sum_{l=1}^s p_{kl}^j \text{ for all } s = 1, \dots, z.$$

Let us assume that the decision-maker adds additional information and indicates that the outcome is improved more by switching from e_l to e_{l+1} than from e_{l+1} to e_{l+2} for all $l = 1, \dots, z-2$. In such case Ordinal Sec-

ond Degree Stochastic Dominance (OSSD) rule can be employed:

Definition 4 – Ordinal Second Degree Stochastic Dominance:

$$X_k^i \text{ OSSD } X_k^j \text{ if and only if}$$

$$\sum_{r=l=1}^s \sum_{r=l=1}^r p_{kl}^i \leq \sum_{r=l=1}^s \sum_{r=l=1}^r p_{kl}^j \text{ for all } s = 1, \dots, z.$$

Spector et al (1996) showed that OFSD rule is equivalent to the expected utility rule for all decision-makers preferring larger outcomes, while the OSSD rule is equivalent to the expected utility rule for risk-averse decision-makers preferring larger outcomes.

Thus, stochastic dominance rules can be applied for variables measured on both cardinal and ordinal scale.

5. Multicriteria procedure for a project selection problem

Let us assume that additive utility independence condition is fulfilled. In such case the multiattribute comparison of two alternatives can be decomposed into n one-attribute comparisons. As these comparisons are obtained from SD rules, they are expressed in terms of “ a_i is at least as good as a_j ” in relation to each attribute and for all pairs $(a_i, a_j) \in \mathbf{A} \times \mathbf{A}$. The following question arises: how the SD concept can be implemented in modelling global preferences? Huang et al (1978) proposed Multiattribute Stochastic Dominance (MSD). According to this rule, alternative a_i is at least as good as a_j in the sense of MSD, if and only if the evaluations of alternative a_i dominate corresponding evaluations of a_j according SD rules with respect to all attributes. In practice this rule is rarely verified. Zaras and Martel (1994) suggested weakening the unanimity condition and accepting a majority attribute condition. They proposed MSD_r – multiattribute stochastic dominance for a reduced number of attributes. This approach is based on the observation that people tend to simplify the multiattribute problem by taking into account only the most important attributes.

In this paper a multicriteria procedure based on stochastic dominance and PROMETHEE II methodology is proposed. The procedure consists of the following steps:

1. Simulation analysis of considered projects with respect to financial measures;
2. Collection of experts opinions with respect to qualitative criteria used in project evaluation;
3. Generation of distributional evaluations of projects with respect to attributes;
4. Identification of stochastic dominance relations for all pairs of projects with respect to each attribute;
5. Generation of the final ranking of alternatives using PROMETHEE II methodology.

The first step of our procedure is simulation. Based on the predictions of interest rates, exchange rates, prices

and demands for products, a series of experiments is conducted for each project. Thus sequences of criteria values are obtained for each project. At the same time experts’ opinions on alternative propositions have to be collected. The results of simulation experiments and experts’ evaluations can be used for generating probability distributions.

In the procedure proposed here the evaluations of alternative projects are compared according to stochastic dominance rules. It is assumed that the decision-maker is risk-averse. Such assumption is usually taken in finance and corresponds to the results of experiments of Kahneman and Tversky (1979) showing that decision-makers are usually risk-averse in relation to attributes defined in the domain of gains. As a result, we will use FSD / SSD rules for modelling decision-maker’s preferences with respect to attributes measured on cardinal scale, and OFSD / OSSD rules in the case of attributes measured on ordinal scale.

Classical decision theory considers two situations when two alternatives are compared: strict preference and indifference. Roy (1985) showed that such assumption is groundless and may cause problems in decision aid. He noticed that it is not reasonable to accept strict preference if the evaluations of actions differ insignificantly. Instead, he considers four basic situations: strict preference, weak preference, indifference and incomparability. Indifference and preference thresholds are used to distinguish situations of indifference, weak and strict preference. Veto threshold is also employed to indicate situations when difference between two alternatives with respect to one specified attribute will require the decision-maker to negate any possible outranking relationship indicated by other criteria.

In this paper we will employ preference and veto threshold concepts. We assume that three situations can be considered when preferences are modelled with respect to single attribute: strict preference, weak preference and indifference as was proposed in Nowak (2004).

Let SD denotes stochastic dominance relation (FSD/SSD/OFSO/OSSD). The following situations are distinguished when a_i and a_j are compared with respect to attribute X_k :

1. a_i is strictly preferred to a_j :

$$a_i P a_j \Leftrightarrow X_k^i \text{ SD } X_k^j \text{ and } \mu_k^i \geq \mu_k^j + p_k(\mu_k^j),$$

2. a_j is strictly preferred to a_i :

$$a_j P a_i \Leftrightarrow X_k^j \text{ SD } X_k^i \text{ and } \mu_k^j \geq \mu_k^i + p_k(\mu_k^i),$$

3. a_i is weakly preferred to a_j :

$$a_i Q a_j \Leftrightarrow X_k^i \text{ SD } X_k^j \text{ and } \mu_k^i < \mu_k^j < \mu_k^j + p_k(\mu_k^j),$$

4. a_j is weakly preferred to a_i :

$$a_j Q a_i \Leftrightarrow X_k^j \text{ SD } X_k^i \text{ and } \mu_k^j < \mu_k^i < \mu_k^i + p_k(\mu_k^i),$$

5. non-preference – otherwise.

The final ranking is generated by PROMETHEE II technique (Brans et al, 1986). The procedure includes the following steps:

1. Calculation of concordance indexes for each pair of alternatives (a_i, a_j) :

$$c(a_i, a_j) = \sum_{k=1}^n w_k \Phi_k(a_i, a_j),$$

where weighting coefficients w_k sum up to one, and

$$\Phi_k(a_i, a_j) = \begin{cases} 1 & \text{if } X_k^i \text{ SD } X_k^j \\ & \text{and } \mu_k^i \geq \mu_k^j + p_k(\mu_k^i) \\ \frac{\mu_k^i - \mu_k^j}{p_k(\mu_k^i)} & \text{if } X_k^i \text{ SD } X_k^j \\ & \text{and } \mu_k^j < \mu_k^i < \mu_k^j + p_k(\mu_k^i) \\ 0 & \text{otherwise.} \end{cases}$$

2. Calculation of discordance indexes for each pair of alternatives and for each attribute:

$$d_k(a_i, a_j) = \begin{cases} 1 & \text{if } X_k^j \text{ SD } X_k^i \text{ and} \\ & \mu_k^j > \mu_k^i + v_k(\mu_k^i) \\ \frac{\mu_k^j - \mu_k^i - p_k(\mu_k^i)}{v_k(\mu_k^i) - p_k(\mu_k^i)} & \text{if } F_{jk} \text{ SD}_T F_{ik} \text{ and} \\ & \mu_k^i + p_k(\mu_k^i) < \mu_k^j \leq \mu_k^i + v_k(\mu_k^i) \\ 0 & \text{otherwise.} \end{cases}$$

3. Calculation of credibility indexes:

$$\sigma(a_i, a_j) = c(a_i, a_j) \cdot \prod_{k \in D(a_i, a_j)} \frac{1 - d_k(a_i, a_j)}{1 - c(a_i, a_j)},$$

where

$$D(a_i, a_j) = \{k: d_k(a_i, a_j) > c(a_i, a_j)\}.$$

4. Calculation of outgoing flow $\phi^+(a_i)$ and incoming flow $\phi^-(a_i)$ for each alternative:

$$\phi^+(a_i) = \sum_{j=1}^m \sigma(a_i, a_j)$$

$$\phi^-(a_i) = \sum_{j=1}^m \sigma(a_j, a_i).$$

5. Calculation of net flow $\phi(a_i)$ for each alternative:

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i).$$

6. Generation of final ranking according to descending order of net flows.

6. Numerical example

Let us consider the decision-maker has to choose one of ten projects taking into account four attributes: X_1 – net present value, X_2 – profitability index, X_3 – chances of success, X_4 – level of technological novelty. The evaluations of projects with respect to attributes X_1 and X_2 are obtained employing simulation technique. The evaluations of alternatives with respect to attributes X_3 and X_4 are constructed on the basis of experts' opinions.

In each project an initial investment is made at the beginning of the first period. Cash flows are not known with certainty, but based on past experience and stored data under a variability of economic conditions, analysts estimated probability distributions for cash flows associated with each of the projects. In our example we assume that all of them are uniform distributions. Table 2 shows projects financial characteristics (Tables 2 – 11 are given in appendix).

A series of 1000 simulations have been conducted assuming 30 % discount rate. Average, minimal and maximal values of NPV and PI are presented in Table 3. Results of simulation experiments are exploited for constructing distributional evaluations of actions with respect to attributes X_1 and X_2 – probability equal to 0,001 has been assumed to each observation.

In the next step 10 experts have been asked to evaluate each project with respect to attributes X_3 (chances of success) and X_4 (level of technical novelty). Each of them has been asked to assign mark from 1 (lowest) to 10 (highest). Thus, assuming equal probabilities of 0,1 to each expert's evaluation, distributional evaluations with respect to quality attributes have been obtained (Tables 4, 5).

Once, the knowledge base necessary for the selection procedure has been generated, next steps of the procedure can be realised. First, alternatives are compared with respect to each attribute separately. Stochastic dominance relations between distributional evaluations are shown in Tables 6, 7, 8 and 9.

The last step of the procedure is the construction of the multiattribute ranking. Let us assume that the decision-maker accepted the following weighting coefficients: $w_1 = 0,4$, $w_2 = 0,2$, $w_3 = 0,25$, $w_4 = 0,15$. Thus, he/she finds net present value to be the most important, next are chances of success, profitability index and level of technical novelty. Let us also assume that the discussion with the decision-maker resulted in setting the following values of preference and veto thresholds:

$$\begin{aligned} p_1 &= 3,00 & v_1 &= 10,00 \\ p_2 &= 0,05 & v_2 &= 0,12 \\ p_3 &= 0,50 & v_3 &= 3,00 \\ p_4 &= 0,50 & v_4 &= 3,00 \end{aligned}$$

Thus, it is assumed that alternative a_i is strictly preferred to a_j with respect to attribute X_1 if either FSD or SSD test is positively verified and the difference between average evaluations of a_i and a_j is at least equal to 3.

For attribute X_2 this difference should not be less than 0,05, while for attributes X_3 and X_4 it should be at least 0,5. If the stochastic dominance test is verified, but the differences between average evaluations are less than preference thresholds, then weak preference is assumed. For example, it is assumed that alternative a_1 is strictly preferred to a_2 with respect to attribute X_1 because the evaluation of a_1 with respect to X_1 dominated corresponding evaluation of a_2 (X_1^1 SSD X_1^2) and the difference between average evaluations is greater than 3 ($\mu_1^1 = 131,8263, \mu_1^2 = 124,5909$). Alternative a_5 is weakly preferred to a_1 with respect to X_1 , as X_1^5 FSD X_1^1 , but the difference between average evaluations is less than 3. Non-preference is assumed for alternatives a_1 and a_7 with respect to X_1 because neither X_1^1 SD X_1^7 , nor X_1^7 SD X_1^1 .

Concordance indexes are presented in Table 10. As alternative a_1 is strictly preferred to a_2 with respect to attributes X_1 and X_4 , so the value of $c(a_1, a_2)$ is a sum of w_1 and w_4 ($0,4 + 0,15 = 0,55$). Alternative a_5 is preferred to a_1 with respect to all attributes, but as weak preference is assumed for attributes X_1 and X_4 , so $c(a_5, a_1)$ is less than 1.

Calculation of credibility indexes (Table 11) is the next step of the procedure. For some pairs of alternatives values of credibility indexes are the same as corresponding values of concordance indexes and for the rest they are less. For example, $c(a_1, a_4) = 0,5222$, while $\sigma(a_1, a_4) = 0,0000$. This is the result of the large difference between average evaluations of a_4 and a_1 with respect to attributes X_3 and X_4 , which are greater than values of veto thresholds.

Credibility indexes are used for calculating outgoing and incoming flows. The final ranking of actions generated according descending order of net flows is presented in Table 1.

Table 1. Final ranking of projects

Alternative	$\phi+(a_i)$	$\phi-(a_i)$	$\phi(a_i)$
a_7	5,3359	0,0000	5,3359
a_{10}	3,5675	0,6408	2,9267
a_8	3,1638	0,6900	2,4738
a_5	3,0229	1,1472	1,8757
a_9	2,5125	1,6864	0,8261
a_4	0,5858	1,4715	-0,8856
a_6	1,7500	2,9791	-1,2291
a_3	2,0608	3,6397	-1,5788
a_1	0,5500	4,5511	-4,0011
a_2	0,1482	5,8917	-5,7435

The best alternative in the ranking is a_7 , next are a_{10} and a_8 . Alternative a_2 is the worst.

7. Conclusions

The methodology presented in this paper can be successfully employed for comparing various investment

projects. There are several advantages of such an approach. First of all, the decision-maker is able to take into account different criteria. Both quantitative and qualitative criteria can be considered. Next, by using stochastic dominance rules it is possible to take into account the uncertainty factor. Finally, the employment of preference and veto thresholds makes it possible to consider situations when distributional evaluations differ insensibly or the difference between them is very large.

The procedure presented in this paper can be also successfully applied in other fields. Stochastic dominance rules are widely exploited in finance. However applications in such fields as production process control, inventory control or capacity planning can be considered as well.

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INVESTICINIŲ PROJEKTŲ ĮVERTINIMAS MODELIAVIMO IR DAUGIAKRITERINIAIS SPRENDIMŲ PRIĖMIMO METODAIS

M. Nowak

Santrauka

Nagrinėjamas investicinių projektų pasirinkimo uždavinys. Daugeliu atvejų projektų įvertinimas yra daugiakriterinis uždavinio sprendimas. Finansinė analizė labai svarbi, bet techniniai, socialiniai ir ekologiniai veiksniai taip pat turėtų būti įvertinti. Finansiniai rodikliai dažniausiai yra kiekybiniai, kiti nagrinėjami rodikliai gali turėti kokybinius matavimus. Projektai analizuojami neapibrėžtumo sąlygomis, todėl ši uždavinį galima traktuoti kaip stochastinį daugiakriterinį sprendimų priėmimą. Stochastinės nominacijos bei daugiakriterinei sprendimų priėmimo procedūrai atlikti taikomas *PROMETHEE II* metodas. Projektų finansiniams įvertinimams nustatyti taikoma modeliavimo technologija, ekspertų metodai – projektams vertinti kitų kriterijų aspektu. Taigi analizuojami kokybiniai ir kiekybiniai rodikliai.

Raktažodžiai: projektų pasirinkimo uždavinys, daugiakriterinė analizė, modeliavimas neapibrėžtumo sąlygomis, modeliavimas.

Maciej NOWAK is an assistant professor in the Department of Operations Research at the Karol Adamiecki University of Economics in Katowice, Poland. He is a member of the Polish Section of INFORMS (Institute for Operations Research and Management Sciences). His research interests include decision theory, operations research, particularly multiple criteria decision making and simulation.

Appendix

Table 2. Projects financial characteristics

Project	Initial investment EUR (10) ³	Cash flow at the end of year (uniform probability distributions) EUR (10) ³				
		1	2	3	4	5
<i>a</i> ₁	500	[200; 300]	[225; 325]	[250; 350]	[200; 300]	[150; 250]
<i>a</i> ₂	400	[100; 200]	[175; 275]	[200; 300]	[225; 325]	[200; 300]
<i>a</i> ₃	450	[0; 100]	[275; 375]	[275; 375]	[275; 375]	[275; 375]
<i>a</i> ₄	500	[150; 250]	[200; 300]	[275; 375]	[275; 375]	[150; 250]
<i>a</i> ₅	350	[150; 250]	[175; 275]	[175; 275]	[100; 200]	[100; 200]
<i>a</i> ₆	525	[225; 275]	[250; 325]	[250; 325]	[225; 275]	[225; 275]
<i>a</i> ₇	425	[150; 200]	[300; 375]	[300; 375]	[100; 175]	[50; 100]
<i>a</i> ₈	575	[200; 300]	[275; 350]	[300; 375]	[275; 350]	[250; 325]
<i>a</i> ₉	400	[175; 200]	[225; 250]	[225; 250]	[225; 250]	[200; 325]
<i>a</i> ₁₀	375	[200; 225]	[225; 250]	[200; 225]	[175; 200]	[150; 175]

Table 3. Results of simulation experiments

Project	Net present value				Profitability index			
	Average	Minimal	Maximal	St.dev.	Average	Minimal	Maximal	St.dev.
<i>a</i> ₁	131,8263	33,9500	228,4800	35,0000	1,2637	1,0680	1,4570	0,0700
<i>a</i> ₂	124,5909	25,2500	221,2000	32,9221	1,3115	1,0630	1,5530	0,0823
<i>a</i> ₃	128,7827	33,0700	211,5300	33,3998	1,2862	1,0730	1,4700	0,0742
<i>a</i> ₄	116,5585	24,9100	202,0600	33,0418	1,2331	1,0500	1,4040	0,0661
<i>a</i> ₅	132,6550	38,1500	223,5900	33,0726	1,3790	1,1090	1,6390	0,0945
<i>a</i> ₆	121,8794	56,2800	177,5900	20,9614	1,2321	1,1070	1,3380	0,0399
<i>a</i> ₇	130,1713	67,9500	189,1700	21,2575	1,3063	1,1600	1,4450	0,0500
<i>a</i> ₈	141,5941	63,0000	220,0000	29,5711	1,2462	1,1100	1,3830	0,0514
<i>a</i> ₉	131,8910	108,1000	156,6500	8,2891	1,3297	1,2700	1,3920	0,0207
<i>a</i> ₁₀	133,9424	105,7400	158,0100	8,4464	1,3572	1,2820	1,4210	0,0225

Table 4. Distributional evaluations for attribute X_3 (chances of success)

Attribute value	Projects									
	<i>a</i> ₁	<i>a</i> ₂	<i>a</i> ₃	<i>a</i> ₄	<i>a</i> ₅	<i>a</i> ₆	<i>a</i> ₇	<i>a</i> ₈	<i>a</i> ₉	<i>a</i> ₁₀
1										
2	0,2								0,1	0,1
3	0,4	0,2						0,1	0,2	0,2
4	0,2	0,2	0,2		0,1	0,1		0,1	0,3	0,3
5	0,2	0,1	0,4		0,2	0,4	0,1	0,1		0,3
6		0,5	0,2	0,4	0,2	0,2	0,3	0,2	0,2	0,1
7			0,2	0,2	0,3	0,2	0,1		0,2	
8				0,4	0,1		0,2			
9					0,1	0,1	0,1	0,2		
10							0,2	0,3		
Av. evaluation	3,4	4,9	5,4	7,0	6,4	5,9	7,5	7,2	4,6	4,1

Table 5. Distributional evaluations for attribute X_4 (level of technical novelty)

Attribute value	Projects									
	<i>a</i> ₁	<i>a</i> ₂	<i>a</i> ₃	<i>a</i> ₄	<i>a</i> ₅	<i>a</i> ₆	<i>a</i> ₇	<i>a</i> ₈	<i>a</i> ₉	<i>a</i> ₁₀
1		0,3								
2	0,3	0,3			0,1					
3	0,1	0,1			0,2			0,1	0,4	
4	0,1	0,1	0,4		0,3	0,1		0,2	0,3	
5	0,4	0,2			0,1	0,2		0,3		0,1
6	0,1		0,5		0,3	0,2	0,1	0,1		0,3
7			0,1	0,3		0,2	0,2		0,1	0,1
8				0,5		0,1	0,3	0,2	0,2	0,3
9				0,1		0,2	0,2	0,1		0,2
10				0,1			0,2			
Av. evaluation	3,9	2,6	5,3	8,0	4,3	6,6	8,2	5,7	4,7	7,2

Table 9. Stochastic dominance relations for X_4 (level of technical novelty)

	X_4^1	X_4^2	X_4^3	X_4^4	X_4^5	X_4^6	X_4^7	X_4^8	X_4^9	X_4^{10}
X_4^1		OFSD								
X_4^2										
X_4^3	OFSD	OFSD			OFSD				OSSD	
X_4^4	OFSD	OFSD	OFSD		OFSD	OFSD		OFSD	OFSD	OFSD
X_4^5	OSSD	OFSD								
X_4^6	OFSD	OFSD	OFSD		OFSD			OFSD	OFSD	
X_4^7	OFSD	OFSD	OFSD		OFSD	OFSD		OFSD	OFSD	OFSD
X_4^8	OFSD	OFSD			OFSD				OFSD	
X_4^9	OSSD	OFSD								
X_4^{10}	OFSD	OFSD	OFSD		OFSD	OFSD		OFSD	OFSD	

Table 10. Concordance indexes

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_1	0,0000	0,5500	0,0000	0,5222	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
a_2	0,2500	0,0000	0,0000	0,6000	0,0000	0,0000	0,0000	0,0000	0,1500	0,2500
a_3	0,4901	0,8000	0,0000	0,6000	0,1500	0,0000	0,0000	0,0000	0,4000	0,2500
a_4	0,4000	0,4000	0,4000	0,0000	0,4000	0,4000	0,0000	0,1500	0,4000	0,4000
a_5	0,6805	1,0000	0,8500	0,6000	0,0000	0,4500	0,0000	0,0000	0,2500	0,2500
a_6	0,4000	0,4000	0,4000	0,4000	0,1500	0,0000	0,0000	0,1500	0,4000	0,2500
a_7	0,5704	0,8000	0,6655	0,6000	0,4000	1,0000	0,0000	0,5000	0,4000	0,4000
a_8	0,8000	0,8000	0,4000	0,4525	0,5500	0,0000	0,0000	0,0000	0,4000	0,2500
a_9	0,6086	0,6231	0,5742	0,6000	0,0000	0,6000	0,3231	0,2000	0,0000	0,2500
a_{10}	0,8821	0,7328	0,7500	0,6000	0,3217	0,7500	0,6000	0,3500	0,2598	0,0000

Table 11. Credibility indexes

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	Sum
a_1	0,0000	0,5500	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,5500
a_2	0,1194	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0288	0,0000	0,1482
a_3	0,4901	0,8000	0,0000	0,1800	0,0645	0,0000	0,0000	0,0000	0,4000	0,1262	2,0608
a_4	0,0000	0,1858	0,0000	0,0000	0,0000	0,4000	0,0000	0,0000	0,0000	0,0000	0,5858
a_5	0,6805	1,0000	0,8500	0,0000	0,0000	0,2291	0,0000	0,0000	0,2500	0,0133	3,0229
a_6	0,4000	0,4000	0,4000	0,4000	0,0000	0,0000	0,0000	0,1500	0,0000	0,0000	1,7500
a_7	0,5704	0,8000	0,6655	0,6000	0,4000	1,0000	0,0000	0,5000	0,4000	0,4000	5,3359
a_8	0,8000	0,8000	0,4000	0,2315	0,5500	0,0000	0,0000	0,0000	0,3478	0,0346	3,1639
a_9	0,6086	0,6231	0,5742	0,0000	0,0000	0,6000	0,0000	0,0400	0,0000	0,0667	2,5126
a_{10}	0,8821	0,7328	0,7500	0,0600	0,1328	0,7500	0,0000	0,0000	0,2598	0,0000	3,5675
Sum	4,5511	5,8917	3,6397	1,4715	1,1473	2,9791	0,0000	0,6900	1,6864	0,6408	