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CODAS METHOD FOR 2-TUPLE LINGUISTIC PYTHAGOREAN FUZZY MULTIPLE ATTRIBUTE GROUP DECISION MAKING AND ITS APPLICATION TO FINANCIAL MANAGEMENT PERFORMANCE ASSESSMENT

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Abstract. Financial management performance evaluation (FMPE) has a significant effect on the identifying an investment chance. We can usually consider FMPE as a multiple attribute group decision making (MAGDM) issue, and the MAGDM method is needed to address it. Uncertainty may be one of the significant factors which could influence the process of MAGDM. In order to handle the uncertainty of group decision-making issues, MAGDM approaches along with 2-tuple linguistic Pythagorean fuzzy sets (2TLPFSs) have been designed. In this essay, CODAS method is extended to 2TLPFSs to tackle MAGDM issues. Linguistic variables and 2TLPFSs are also used to extend the CODAS method. An application of the presented 2-tuple linguistic Pythagorean fuzzy CODAS (2TLPF-CODAS) method to a case study of FMPE problem with 2-tuple linguistic Pythagorean fuzzy numbers (2TLPFNs) is given. To confirm the results, a comparative analysis between the fuzzy CODAS and 2TLPF-TODIM is performed. The results of the comparison illustrate that the presented 2TLPF-CODAS method offers effective and steady results.

Keywords: MAGDM, 2TLPFSs, CODAS method, 2TLPF-CODAS method, financial management performance evaluation.

JEL Classification: C43, C61, D81.

Introduction

The numbers 0 and 1 are used to deliver the "no" and "yes" of the depiction of the thing in the exact mathematical set, but there is often an ambiguous state in the depiction of the real world. On the basis of this, (Zadeh, 1965) presented the theory of fuzzy set which used the membership degree to describe things' ambiguity, but it fail to depict both support and opposition ideas (Wei, 2019a, 2019b; Wu et al., 2019a, 2019b; Wu et al., 2018). Thus,

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Atanassov (1986) designed the intuitionistic fuzzy sets (IFSs) which could conquer this limitation. Reformat and Yager (2014) extended the IFSs with the Pythagorean fuzzy sets (PFSs), $u(x)$ and $v(x)$ should meet the new condition that $u^2(x)+v^2(x)\leq 1$, thus extending the description range of the IFSs. Until now, PFSs have widely applied in MADM and MAGDM (Khan et al., 2019; Zeb et al., 2019; Zhu et al., 2018). Based on the PFSs, Reformat and Yager (2014) presented a novel recommendation system which was collaborative oriented. Peng and Yang (2015) studied Pythagorean fuzzy numbers' division and subtraction algorithms. Garg (2017a) researched the confidence level's statistical concept into the PFSs. Ren et al. (2016) extended the Pythagorean fuzzy TODIM method on the basis of the prospect theory. Garg (2017b) improved scoring function calculation method for PFNs. Zeng et al. (2016) connected the distance measure with the PFSs. Li et al. (2018) defined some operators of Pythagorean Fuzzy Hamy Mean to address MAGDM issues. Zhang et al. (2017) combined the generalized Bonferroni mean with PFNs. Garg (2016) linked the PFSs with the Einstein operator. Li and Lu (2019) proposed some similarity and distance measures under PFSs. Wang et al. (2019b) designed the generalized Dice similarity measures for MAGDM with PFNs. Zhang (2016) extended the PFSs to the form of interval PFSs. Zhang and Jiang (2010) designed entropy for PFNs. Wei (2019c) defined the Hamacher power operators for PFNs. Zhang et al. (2016) presented a model which was about rough set under PFSs by means of multi-granular rough set. Deng et al. (2018a) gave the concept of 2TLPFSs and proposed various Hamy mean operations under 2TLPFSs. Deng et al. (2018) developed some Bonferroni mean operations under 2TLPFSs.

The CODAS method was defined by Keshavarz Ghorabaee et al. (2016). Panchal et al. (2017) employed integrated MCDM framework on the basis of AHP and CODAS method. Badi et al. (2018) made use of CODAS approach to choose the desalination plant's best location in Libya's northwestern coast. Ghorabaee et al. (2018) extended the CODAS method to fuzzy environment to choose the most desirable suppliers. Pamucar et al. (2018) introduced new CODAS method with linguistic Neutrosophic Numbers (LNN).

Therefore, the above research failed to concern about the MAGDM issue with 2TLPFNs in terms of CODAS approach. In this essay, we utilize the 2TLPFNs to expand the CODAS method to design a novel MAGDM method. An example is used to display the proposed model's applicability. To illustrate the 2TLPF-CODAS method's stability, we make a comparison between 2TLPF-CODAS method & 2TLPF-TODIM method (Deng & Gao, 2019). The calculating results demonstrate that the presented approach is stability and validity.

This paper's remainder is arranged subsequently. Some fundamental concepts of P2TLSs are given in Section 1. The CODAS method is built to handle MAGDM issues with 2TLPFNs in Section 2. A case study for FMPE is given to illustrate the presented approach in Section 3. In last section, the essay is made a conclusion.

1. Preliminaries

The fundamental concepts of 2-tuple linguistic sets (2TLSs) (Herrera & Martinez, 2001a), Pythagorean fuzzy sets (PFSs) (Reformat & Yager, 2014) and 2TLPFSs (Deng et al., 2018a) are given in this chapter.

1.1. 2TLSs

Definition 1 (Herrera & Martinez, 2001b). $S=\{s_i \mid i=0,1,\ldots,t\}$ is designed to be a linguistic term set (LTS) with odd integer. s_i was employed to depict the possible value in a LTS, and the set S could be depicted as:

$$S=\left\{\begin{array}{l} s_0=\text{extremely poor}, s_1=\text{very poor}, s_2=\text{poor}, s_3=\text{medium}, \\ s_4=\text{good}, s_5=\text{very good}, s_6=\text{extremely good}. \end{array}\right\}.$$

1.2. PFSs

The PFSs A in a ordinary fixed set X can be depicted underneath (Reformat & Yager, 2014):

$$A=\left\{\left\langle x, \mathrm{u}_A(x), v_A(x)\right\rangle \mid x \in X\right\}, \tag{1}$$

which $u_A(x)$ and $v_A(x)$ denoted the membership degree and the non-membership degree, which meet such condition: $0 \le u_A(x) \le 1, 0 \le v_A(x) \le 1$ and $\left(u_A(x)\right)^2+\left(v_A(x)\right)^2 \le 1$.

1.3. 2TLPFSs

Deng et al. (2018a) gave the definition of 2TLPFSs.

Definition 1. (Deng et al., 2018a). Suppose that $P=\{p_0, p_1, \ldots, p_t\}$ is a LTSs with odd integer $t=1$. If $\left(s_\alpha(x), \varphi(x)\right), \left(s_\beta(x), \vartheta(x)\right)$ is defined for $s_\alpha(x), s_\beta(x) \in P$, $\varphi(x), \vartheta(x) \in [-0.5, 0.5)$, where $\left(s_\alpha(x), \varphi(x)\right), \left(s_\beta(x), \vartheta(x)\right)$ depict the membership and non-membership by 2TLSs, then the definition of 2TLPFSs could be defined:

$$P=\left[\!\left[\left\langle x, \left\{\left(s_\alpha(x), \varphi(x)\right), \left(s_\beta(x), \vartheta(x)\right)\right\}\right\rangle \mid x \in X\right]\!\right], \tag{2}$$

where $0 \le \Delta^{-1}\left(s_\alpha(x), \varphi(x)\right) \le t, 0 \le \Delta^{-1}\left(s_\beta(x), \vartheta(x)\right) \le t$,
$0 \le \left(\Delta^{-1}\left(s_\alpha(x), \varphi(x)\right)\right)^2+\left(\Delta^{-1}\left(s_\beta(x), \vartheta(x)\right)\right)^2 \le t^2$.

In order to easy computation, $p=\left\{\left(s_\alpha, \varphi\right), \left(s_\beta, \vartheta\right)\right\}$ denotes the 2TLPFN.

Then, 2TLPFNs' score and accuracy function are shown as follows:

Definition 2. (Deng et al., 2018a). Let $p=\left\{\left(s_\alpha, \varphi\right), \left(s_\beta, \vartheta\right)\right\}$ be a 2TLPFN in P. Then the score and accuracy functions of p are defined as follows:

$$S(p)=\Delta\left\{\frac{t}{2}\left(1+\left(\frac{\Delta^{-1}\left(s_\alpha, \varphi\right)}{t}\right)^2-\left(\frac{\Delta^{-1}\left(s_\beta, \vartheta\right)}{t}\right)^2\right)\right\}, S(p) \in [0, t]; \tag{3}$$

$$H(p)=\Delta\left\{t\left(\left(\frac{\Delta^{-1}\left(s_\alpha, \varphi\right)}{t}\right)^2+\left(\frac{\Delta^{-1}\left(s_\beta, \vartheta\right)}{t}\right)^2\right)\right\}, H(p) \in [0, t]. \tag{4}$$

Then, Deng et al. (2018a) gave some novel operations on the 2TLPFNs.

Definition 3. (Deng et al., 2018a). Let $p_1 = \left\{ \left(s_{\alpha_1}, \varphi_1\right), \left(s_{\beta_1}, \vartheta_1\right) \right\}$ and $p_2 = \left\{ \left(s_{\alpha_2}, \varphi_2\right), \left(s_{\beta_2}, \vartheta_2\right) \right\}$ be two 2TLPFNs, then

$$(1)\ p_1 \oplus p_2 = \left\{ \Delta\left(t\sqrt{1-\left(1-\left(\frac{\Delta^{-1}\left(s_{\alpha_1},\varphi_1\right)}{t}\right)^2\right)\left(1-\left(\frac{\Delta^{-1}\left(s_{\alpha_2},\varphi_2\right)}{t}\right)^2\right)} \right), \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{\beta_1},\vartheta_1\right)}{t} \bullet \frac{\Delta^{-1}\left(s_{\beta_2},\vartheta_2\right)}{t} \right)\right) \right\};$$

$$(2)\ p_1 \otimes p_2 = \left\{ \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{\alpha_1},\varphi_1\right)}{t} \bullet \frac{\Delta^{-1}\left(s_{\alpha_2},\varphi_2\right)}{t} \right)\right), \Delta\left(t\sqrt{1-\left(1-\left(\frac{\Delta^{-1}\left(s_{\beta_1},\vartheta_1\right)}{t}\right)^2\right)\left(1-\left(\frac{\Delta^{-1}\left(s_{\beta_2},\vartheta_2\right)}{t}\right)^2\right)} \right) \right\};$$

$$(3)\ \lambda p_1 = \left\{ \Delta\left(t\sqrt{1-\left(1-\left(\frac{\Delta^{-1}\left(s_{\alpha_1},\varphi_1\right)}{t}\right)^2\right)^{\lambda}} \right), \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{\beta_1},\vartheta_1\right)}{t}\right)^{\lambda} \right) \right\};$$

$$(4)\ \left(p_1\right)^{\lambda} = \left\{ \Delta\left(t\left(\frac{\Delta^{-1}\left(s_{\alpha_1},\varphi_1\right)}{t}\right)^{\lambda} \right), \Delta\left(t\sqrt{1-\left(1-\left(\frac{\Delta^{-1}\left(s_{\beta_1},\vartheta_1\right)}{t}\right)^2\right)^{\lambda}} \right) \right\}.$$

Definition 4. (Deng & Gao, 2019). Let $p_1 = \left\{ \left(s_{\alpha_1}, \varphi_1\right), \left(s_{\beta_1}, \vartheta_1\right) \right\}$ and $p_2 = \left\{ \left(s_{\alpha_2}, \varphi_2\right), \left(s_{\beta_2}, \vartheta_2\right) \right\}$ be two 2TLPFNs, then the normalized Hamming distance *HD* is designed:

$$HD\left(p_1, p_2\right) = \frac{1}{2t}\left(\left| \Delta^{-1}\left(s_{\alpha_1},\varphi_1\right) - \Delta^{-1}\left(s_{\alpha_2},\varphi_2\right) \right| + \left| \Delta^{-1}\left(s_{\beta_1},\vartheta_1\right) - \Delta^{-1}\left(s_{\beta_2},\vartheta_2\right) \right| \right). \quad (5)$$

Definition 5. Let $p_1 = \left\{ \left(s_{\alpha_1}, \varphi_1\right), \left(s_{\beta_1}, \vartheta_1\right) \right\}$ and $p_2 = \left\{ \left(s_{\alpha_2}, \varphi_2\right), \left(s_{\beta_2}, \vartheta_2\right) \right\}$ be two 2TLPFNs, then the normalized Hamming distance *ED* is designed:

$$ED\left(p_1, p_2\right) = \sqrt{\frac{1}{2}\left(\left(\frac{\left| \Delta^{-1}\left(s_{\alpha_1},\varphi_1\right) - \Delta^{-1}\left(s_{\alpha_2},\varphi_2\right) \right|}{t} \right)^2 + \left(\frac{\left| \Delta^{-1}\left(s_{\beta_1},\vartheta_1\right) - \Delta^{-1}\left(s_{\beta_2},\vartheta_2\right) \right|}{t} \right)^2 \right)}. \quad (6)$$

2. The CODAS method for MAGDM with 2TLPFNs

The subsequently assumptions or notations are utilized to denote the MAGDM issues with 2TLPFNs. Assume $A=\{A_1,A_2,\cdots,A_m\}$ be some chosen alternatives and $G=\{G_1,G_2,\cdots,G_n\}$ be some designed attributes with weight vector $w=(w_1,w_2,\cdots,w_n)$, where $w_j\in[0,1]$, $\sum_{j=1}^{n}w_j=1$, and some experts $E=\{E_1,E_2,\cdots,E_q\}$ with weight vector $v=(v_1,v_2,\cdots,v_q)$, where $v_k\in[0,1]$, $k=1,2,\cdots,q$, $\sum_{k=1}^{q}v_k=1$. Suppose that there are n qualitative attribute $G=\{G_1,G_2,\cdots,G_n\}$ and their values are assessed by each expert and depicted as linguistic expressions $l_{ij}^k\ (i=1,2,\cdots,m,j=1,2,\cdots,n,k=1,2,\cdots,q)$ in Table 1.

Table 1. Linguistic variables and their 2TLPFNs

Linguistic variable	2TLPFNs
Very low (VL)	$\{(s_0,0),(s_6,0))\}$
Low (L)	$\{(s_1,0),(s_5,0)\}$
Medium low (ML)	$\{(s_2,0),(s_4,0)\}$
Medium (M)	$\{(s_3,0),(s_3,0)\}$
Medium high (MH)	$\{(s_4,0),(s_2,0)\}$
High (H)	$\{(s_5,0),(s_1,0)\}$
Very high (VH)	$\{(s_6,0),(s_0,0)\}$

Then, an extended CODAS method with 2TLPFNs is proposed to tackle the MAGDM issues. The calculating steps are involved as follows:

Step 1. Switch the linguistic information l_{ij}^k into 2TLPFNs $r_{ij}^k=\left\{\left(s_{\phi_{ij}^k},\varphi_{ij}^k\right),\left(s_{\theta_{ij}^k},\vartheta_{ij}^k\right)\right\}$.

Step 2. According to 2TLPFN $r_{ij}^k=\left\{\left(s_{\phi_{ij}^k},\varphi_{ij}^k\right),\left(s_{\theta_{ij}^k},\vartheta_{ij}^k\right)\right\}$ and 2TLPFWA operator (Deng et al., 2018b), the experts' individual evaluations can be fused into the collective 2TLPFNs $r_{ij}=\left\{\left(s_{\phi_{ij}},\varphi_{ij}\right),\left(s_{\theta_{ij}},\vartheta_{ij}\right)\right\}$.

$$R=\left[r_{ij}\right]_{m\times n}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}; \tag{18}$$

$$r_{ij}=2TLPFWA\left(r_{ij}^1,r_{ij}^2\cdots,r_{ij}^q\right)=\bigoplus_{k=1}^{q}\eta_k r_{ij}^k=$$
$$\left\{\Delta\left(t\sqrt{1-\prod_{k=1}^{q}\left(1-\left(\frac{\Delta^{-1}\left(s_{\phi_{ij}^k},\varphi_{ij}^k\right)}{t}\right)^2\right)^{\eta_k}}\right),\Delta\left(t\prod_{k=1}^{q}\left(\frac{\Delta^{-1}\left(s_{\theta_{ij}^k},\vartheta_{ij}^k\right)}{t}\right)^{\eta_k}\right)\right\}. \tag{19}$$

Step 3. Calculate the 2TLPF weighted matrix.

$$t_{ij} = w_j \otimes r_{ij}, \tag{20}$$

where w_j means the attribute weight of G_j, and $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$.

Step 4. Get the negative ideal solution with score and accuracy functions of 2TLPFNs (if score functions are equal, the accuracy functions are chosen to rank the 2TLPFNs):

$$NIS = \left[NIS_j \right]_{1\times n}; \tag{21}$$

$$NIS_j = \min_i S\left(t_{ij}\right). \tag{22}$$

Step 5. Determine the weighted ED_i and HD_i:

$$ED_i = \sum_{j=1}^{n} ED\left(t_{ij}, NIS_j\right); \tag{23}$$

$$HD_i = \sum_{j=1}^{n} HD\left(t_{ij}, NIS_j\right). \tag{24}$$

Step 6. Build the relative assessment matrix RA in subsequently equations:

$$RA = \left[h_{ik} \right]_{m\times m}; \tag{25}$$

$$h_{ik} = \left(ED_i - ED_k\right) + \left(g\left(ED_i - ED_k\right) \times \left(HD_i - HD_k\right)\right), \tag{26}$$

where $k \in \{1, 2, \ldots, m\}$ and g means an important function which could be designed:

$$g(\theta) = \begin{cases} 1 & if\,|\theta| \ge \tau \\ 0 & if\,|\theta| < \tau \end{cases}, \tag{27}$$

where $\tau \in [0.01, 0.05]$ given by DMs. In current study, $\tau = 0.02$.

Step 7. Derive the AS_i by Eq. (28).

$$AS_i = \sum_{k=1}^{m} h_{ik}. \tag{28}$$

Step 8. All the alternatives can be ranked on the basis of the computing results of AS_i.The best alternative has the highest assessment score.

3. Case study and comparative analysis

The financial management performance issue is a classical MAGDM issue (Erdogan et al., 2019; Lu et al., 2019; Roy et al., 2019; Tabatabaei et al., 2019; Wang et al., 2019a; Wang, 2019; Wei et al., 2019a, 2019b). In this chapter, we shall give a case study of the financial management performance to choose the most desirable enterprise which has the best financial performance by utilizing CODAS method with 2TLPFNs. Assume that an enterprise identified an investment chance with enterprise financial performances, and in order to maximize the expected profit, we need to determine the enterprise financial performances of the five enterprises so as to choose the optimal one. The investment company has to make a decision in terms of the subsequently four benifical attributes: G_1 is the enterprise innovation ability; G_2 is the enterprise resource utilization capability; G_3 is the internal process; G_4 is the corporate credit rating. There are five potential enterprises $A_i\left(i=1,2,3,4,5\right)$ to be assessed by using linguistic variables which are listed in Table 1. These linguistic variables are given by the invited DMs $D_k\left(k=1,2,3\right)$ (whose weighting vector $\nu=\left(0.20,0.50,0.30\right)$) within the mentioned attributes (whose weighting vector $\omega=\left(0.20,0.30,0.40,0.10\right)^T$), and set up three decision matrixes respectively as follows $R_k=\left(r_{ij}^{(k)}\right)_{5\times 4}$ $\left(k=1,2,3\right)$ which are recorded in Tables 2–4 respectively.

Table 2. Linguistic assessing matrix by first expert

	G_1	G_2	G_3	G_4
A_1	L	MH	M	VL
A_2	ML	L	VL	MH
A_3	ML	MH	M	M
A_4	H	VH	VH	ML
A_5	MH	M	L	L

Table 3. Linguistic assessing matrix by second expert

	G_1	G_2	G_3	G_4
A_1	MH	MH	M	H
A_2	L	M	VL	M
A_3	H	ML	M	ML
A_4	VH	MH	H	VH
A_5	VL	H	ML	MH

Table 4. Linguistic assessing matrix by third expert

	G_1	G_2	G_3	G_4
A_1	H	M	ML	H
A_2	L	MH	H	L
A_3	MH	L	M	ML
A_4	H	M	VH	MH
A_5	ML	ML	MH	H

Following that, the developed approach is utilized to assess financial management performance of five possible enterprises.

Step 1. Transform the linguistic decision matrixes which are recorded in Tables 2–4 into 2TLPF decision matrix. The results are recorded in Tables 5–7.

Table 5. The assessing matrix with 2TLPFNs by first expert

	G_1	G_2	G_3	G_4
A_1	$\{(s_1,0),(s_5,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_0,0),(s_6,0)\}$
A_2	$\{(s_2,0),(s_4,0)\}$	$\{(s_1,0),(s_5,0)\}$	$\{(s_0,0),(s_6,0)\}$	$\{(s_4,0),(s_2,0)\}$
A_3	$\{(s_2,0),(s_4,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_3,0),(s_3,0)\}$
A_4	$\{(s_5,0),(s_1,0)\}$	$\{(s_6,0),(s_0,0)\}$	$\{(s_6,0),(s_0,0)\}$	$\{(s_2,0),(s_4,0)\}$
A_5	$\{(s_4,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_1,0),(s_5,0)\}$	$\{(s_1,0),(s_5,0)\}$

Table 6. The assessing matrix with 2TLPFNs by second expert

	G_1	G_2	G_3	G_4
A_1	$\{(s_4,0),(s_2,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_5,0),(s_1,0)\}$
A_2	$\{(s_1,0),(s_5,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_0,0),(s_6,0)\}$	$\{(s_3,0),(s_3,0)\}$
A_3	$\{(s_5,0),(s_1,0)\}$	$\{(s_2,0),(s_4,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_2,0),(s_4,0)\}$
A_4	$\{(s_6,0),(s_0,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_5,0),(s_1,0)\}$	$\{(s_6,0),(s_0,0)\}$
A_5	$\{(s_0,0),(s_6,0)\}$	$\{(s_5,0),(s_1,0)\}$	$\{(s_2,0),(s_4,0)\}$	$\{(s_4,0),(s_2,0)\}$

Table 7. The assessing matrix with 2TLPFNs by third expert

	G_1	G_2	G_3	G_4
A_1	$\{(s_5,0),(s_1,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_2,0),(s_4,0)\}$	$\{(s_5,0),(s_1,0)\}$
A_2	$\{(s_1,0),(s_5,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_5,0),(s_1,0)\}$	$\{(s_1,0),(s_5,0)\}$
A_3	$\{(s_4,0),(s_2,0)\}$	$\{(s_1,0),(s_5,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_2,0),(s_4,0)\}$
A_4	$\{(s_5,0),(s_1,0)\}$	$\{(s_3,0),(s_3,0)\}$	$\{(s_6,0),(s_0,0)\}$	$\{(s_4,0),(s_2,0)\}$
A_5	$\{(s_2,0),(s_4,0)\}$	$\{(s_2,0),(s_4,0)\}$	$\{(s_4,0),(s_2,0)\}$	$\{(s_5,0),(s_1,0)\}$

Step 2. According to Tables 5–7 and Eq. (19), the experts' individual evaluations can be fused into the collective assessing matrix with 2TLPFNs (Table 8).

Table 8. Collective assessing matrix with 2TLPFNs

	G_1	G_2	G_3	G_4
A_1	$\{(s_4,0.16),(s_2,-0.05)\}$	$\{(s_4,-0.24),(s_2,0.26)\}$	$\{(s_3,-0.25),(s_3,0.27)\}$	$\{(s_5,-0.30),(s_1,0.43)\}$
A_2	$\{(s_1,0.27),(s_5,-0.22)\}$	$\{(s_3,0.16),(s_3,-0.06)\}$	$\{(s_3,0.28),(s_4,-0.49)\}$	$\{(s_3,-0.08),(s_3,0.22)\}$

End of Table 8

	G_1	G_2	G_3	G_4
A_3	$\{(s_4,0.44),(s_2,-0.38)\}$	$\{(s_2,0.47),(s_4,-0.28)\}$	$\{(s_3,0.00),(s_3,0.00)\}$	$\{(s_2,0.25),(s_4,-0.22)\}$
A_4	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$
A_5	$\{(s_2,0.26),(s_4,0.26)\}$	$\{(s_4,0.23),(s_2,-0.11)\}$	$\{(s_3,-0.22),(s_3,0.40)\}$	$\{(s_4,0.16),(s_2,-0.05)\}$

Step 3. Compute the weighted assessing matrix with 2TLPFNs (Table 9).

Table 9. Collective weighted assessing matrix with 2TLPFNs

	G_1	G_2	G_3	G_4
A_1	$\{(s_2,0.10),(s_5,-0.21)\}$	$\{(s_2,0.23),(s_4,0.48)\}$	$\{(s_2,-0.20),(s_5,-0.29)\}$	$\{(s_2,-0.20),(s_5,0.20)\}$
A_2	$\{(s_1,-0.43),(s_6,-0.27)\}$	$\{(s_2,-0.17),(s_5,-0.15)\}$	$\{(s_2,0.18),(s_5,-0.16)\}$	$\{(s_1,-0.02),(s_6,-0.36)\}$
A_3	$\{(s_2,0.30),(s_5,-0.38)\}$	$\{(s_1,0.39),(s_5,0.20)\}$	$\{(s_2,-0.02),(s_5,-0.45)\}$	$\{(s_1,-0.26),(s_6,-0.27)\}$
A_4	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$	$\{(s_6,0.00),(s_0,0.00)\}$
A_5	$\{(s_1,0.04),(s_5,-0.40)\}$	$\{(s_3,-0.41),(s_4,0.24)\}$	$\{(s_2,-0.18),(s_5,-0.22)\}$	$\{(s_2,-0.49),(s_5,0.36)\}$

Step 4. Obtain the NIS by Eq. (22).The calculating results are recorded in Table 10.

Table 10. NIS with 2TLPFNs

G_1	G_2	G_3	G_4
$\{(s_1,-0.43),(s_6,-0.27)\}$	$\{(s_1,0.39),(s_5,0.20)\}$	$\{(s_2,0.18),(s_5,-0.16)\}$	$\{(s_1,-0.26),(s_6,-0.27)\}$

Step 5. Compute the ED_i and HD_i:

$$ED_1 = 0.4915, ED_2 = 0.1406, ED_3 = 0.2749, ED_4 = 3.4141, ED_5 = 0.3384,$$
$$HD_1 = 0.4766, HD_2 = 0.1292, HD_3 = 0.2690, HD_4 = 3.4098, HD_5 = 0.3240.$$

Step 6. Compute the *RA* matrix (Table 11).

Table 11. Relative assessment matrix (RA)

	A_1	A_2	A_3	A_4	A_5
A_1	0.0000	0.6869	0.4183	–5.8602	0.2913
A_2	–0.7132	0.0000	–0.2800	–6.5584	–0.4069
A_3	–0.4391	0.2627	0.0000	–6.2844	–0.1329
A_4	5.8410	6.5427	6.2741	0.0000	6.1472
A_5	–0.3206	0.3812	0.1126	–6.1658	0.0000

Step 7. Compute the value of AS_i by utilizing Eq. (28).

$$AS_1 = -4.4637, AS_2 = -7.9585, AS_3 = -6.5937, AS_4 = 24.8050, AS_5 = -5.9926.$$

Step 8. In terms of the computing results of AS_i, all the alternatives can be ranked. Evidently, the order is $A_4 > A_1 > A_5 > A_3 > A_2$ and A_4 is the best one among five alternatives.

To indicate this method's validity, it is compared with the 2TLPF-TODIM method's result. The order of 2TLPF-TODIM is also $A_4 > A_1 > A_5 > A_3 > A_2$. As can be seen, the 2TLPF-CODAS method's ranking result is totally consistent with 2TLPF-TODIM method. What's more, two distance formulas' combination is used in 2TLPF-CODAS method, which is more exact than the single one.

Conclusions

Financial management performance evaluation has a significant effect on the identifying an investment chance. Because this process can be regarded as a MAGDM issue, it is necessary to utilize an efficient MAGDM method for it. Besides, since the group decision-making process is within uncertain environment, it makes this assessment complex. In this paper, the expanding CODAS method has been developed to tackle MAGDM issues under 2TLPFNs. The weighted Euclidean and weighted Hamming distances of 2TLPFNs have been employed to decide the alternatives' desirability with regard to a negative-ideal solution. Also, we extend the crisp CODAS method utilizing the linguistic variables which are defined by 2TLPFNs. In the developed 2TLPF-CODAS method, the application of an example of financial management performance assessment problem is given. The comparative analysis demonstrates that the 2TLPF-CODAS method is effective and practical with 2TLPF-TODIM method. For further researches, the proposed method's application will be conducted in many other unpredictable and ambiguous environments.

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