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THE OPTIMAL PROPORTION OF STATE-OWNED SHARES IN AN INDUSTRY CHAIN

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Abstract. This study focuses on the mixed-ownership reform of state-owned enterprises in an industry chain. By constructing an oligopoly model considering the proportion of state-owned shares and product differentiation, this study examines the impacts of the balance of state-owned shares and product differentiation in four scenarios, and analyzes the proportion of state-owned shares to maximize the social welfare of the industry chain. The results reveal a synergistic relationship between the balance of state-owned shares and product differentiation. The equilibrium results vary in different industry chain links and competition modes. The implementation of mixed ownership by upstream enterprises will help improve the overall efficiency of the industry chain. Complete nationalization may be optimal and an upstream monopoly can be realized under certain conditions.

Keywords: industry chain, mixed ownership, the proportion of state-owned shares, share cost.

JEL Classification: L13, L51, H23.

Introduction

Since the Thatcher government's privatization reform in 1979, a wave of privatization reforms of state-owned enterprises has been ongoing worldwide (Scott-Samuel et al., 2014). Some countries have not blindly pursued privatization in this process and have continuously explored suitable methods for their conditions. In recent years, state-owned enterprise reforms in China have grown from early pilot reforms to mid-term property rights and mixed-ownership reforms. The current mixed-ownership reform is a diversified capital structure and has

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the characteristics of state-owned economic dominance and two-way mixing. State-owned and private economies have been closely integrated and have played essential roles in many mixed-economy and mixed-ownership enterprises in China. Mixed-ownership reforms have been implemented in many areas in China, such as the photovoltaics, hydropower, smart grids, and environmental protection industries. Furthermore, the industry is actively developing towards green and cooperative development that relies on intelligent technology; the role of the government is also changing. Whether and how to support industry development requires the government to make decisions based on industrial theoretical analysis and practical results (Chen et al., 2022; Dolgui et al., 2019; Saraji et al., 2021), and mixed ownership is an effective policy. For example, mixed-ownership reform in China focuses on improving and rationally distributing key links in the industry chain to maintain the continuous stability of the industry chain. The mixed reform of state-owned enterprises is combined with the adjustment of the industry layout and strategic reorganization to improve pertinence and orientation. Therefore, this study combines mixed ownership with the industrial chain and analyzes the proportion of state-owned shares to maximize the social welfare of the industry chain.

There is much literature on mixed ownership (Guan et al., 2021; Liu et al., 2019), but there are still some issues worthy of attention in the process of the mixed-ownership reform of state-owned enterprises. Is there an optimal proportion of state-owned shares for improving the social welfare of the industry chain? If so, which factors affect this proportion? Is it necessary for a vertical industry chain to implement mixed ownership? If implemented, which link is chosen? To answer these questions, this study sets up an oligopoly model in the industry chain, analyzes the effect of state-owned shares and product differentiation under different scenarios, and provides guidance on the mixed-ownership reform of state-owned enterprises.

The remainder of this paper is organized as follows. Section 1 presents the literature review. Section 2 constructs an oligopoly model that considers share cost difference and product differentiation. Section 3 examines the equilibrium results of the four situations (Model MPP, Model PMM, Model PMP, and Model PPP), analyzes product differences and the proportion of state-owned shares in the equilibrium results, and conducts a comparative analysis. The main conclusions are presented in the last Section.

1. Literature review

The proportion of state-owned shares, especially the optimal balance, has always been a hot topic in academia. Some scholars believe that nationalization is beneficial and investigate its mechanism. From the perspective of social welfare, Ebina and Shimizu (2019) found that complete nationalization is optimal when the products are homogeneous, similar to the conclusion of Jain and Pal (2012). Carson et al. (2020) studied the nationalization of electric power enterprises, suggesting that nationalization is an outcome of specific institutional and policy conditions, and note that state-owned enterprises have apparent advantages in the market. In general, nationalization has the characteristics of eliminating negative externalities, remedying market failures, and dominating essential areas and crucial links, ensuring their efficiency (Warner & Bel, 2008; Xu et al., 2017; Boubakri et al., 2018). Therefore, some believe that even if it is not fully nationalized, proper control of state-owned capital is required to maintain economic stability (Chen et al., 2018; D' Souza & Nash, 2017; Ohnishi, 2011). However, some scholars hold different views. They believe that privatization is more beneficial for the development of enterprises and society (Boubakri et al., 2017). The efficiency of state-owned enterprises is lower, and profitability is weaker, which perhaps does not conform to the principle of utility maximization (Fang et al., 2017).

Moreover, due to information asymmetry and principal–agent problems, corruption is easily generated in state-owned enterprises, resulting in the misallocation of resources and even waste; privatization can effectively deal with these problems. Kroll and Kou (2019) researched the innovation input and equity of Chinese enterprises and found that introducing non-state capital can significantly increase the input and effect of innovation and bring about an increase in supervision and governance effects. Bachiller (2017) analyzed the impact of privatization on improving performance and believes that the profit-seeking nature of private capital is more likely to highlight short-term returns, gradually improving market competitiveness and production efficiency (Chen, 2017). Simultaneously, some scholars believe that nationalization, privatization, and mixed ownership can be optimal (Chen et al., 2019a; Bárcena-Ruiz & Garzón, 2020). The choice of the specific proportion of state-owned shares is affected by various factors, such as the size and cost of the enterprises, the status of corporate social responsibility, market structure, and the legal environment (Chen et al., 2019b; Kim et al., 2019; Pagliari & Graham, 2019).

Recent literature on industry chains in state-owned enterprises is abundant. Most scholars believe that the optimization of the ownership structure of state-owned enterprises has enhanced the overall social welfare of the industry chain and improved governance and innovation efficiency (Aghion et al., 2013; Zhang et al., 2020). Wu et al. (2016) analyzed the mixed oligopoly model of industry chain ownership and found that downstream privatization plays a positive role in the development of the entire industry chain. Wu and Ma (2015) introduced dynamics into the analysis of ownership in the industry chain and found that a partially privatized multi-product development model is more effective. Saha and Sensarma (2011) started with the management incentive problem in implementing mixed ownership, aiming to improve the internal governance efficiency of mixed-ownership enterprises and avoid principal–agent issues. Chang and Chen (2016) conclude that if the market is large or the output of competitors is small, the government should subsidize downstream private companies. These studies provide evidence of the persistence of state-owned enterprises in specific industries.

Compared to existing research, the contributions of this study are as follows. First, there is insufficient literature on the optimal state-owned share proportion of mixed-ownership enterprises in vertical industry chains (Sato & Matsumura, 2019). By constructing a vertical oligopoly model, we compare and analyze the feasibility and effectiveness of mixed ownership in different links of the industry chain, which has significant implications for investigating policy effects. Second, many existing models do not simultaneously consider cost differences and product differentiation. Cost differences are an essential factor affecting enterprises' production and operational activities (Matsumura & Okamura, 2015). Moreover, product differentiation is necessary for enterprises to compete and satisfy increasingly diverse consumer needs (Dong et al., 2018; Saha, 2009). Although a few studies involve cost differences and product differentiation, they rarely analyze the optimal proportion of state-owned shares in mixed-ownership enterprises (Lambertini & Pignataro, 2019). What roles do cost and product differentiation play in determining the optimal balance of state-owned shares? What impacts do they have on enterprises, consumers, and the government? These problems require further analysis and research. For this reason, this study incorporates share costs into the cost function, assuming that the prices of different types of shares and various industry chain links are different, while simultaneously considering the impact of product differentiation.

Third, the optimal proportion of state-owned shares has always been controversial. We focus on the equity design of state-owned enterprises in the industry chain and use the Cournot and Stackelberg models to analyze the roles of the proportion of state-owned shares and solve the optimal balance of state-owned shares under different scenarios. This has specific guiding significance for the selection of corporate equity structure and the formulation of government state-owned enterprise reform policies in China.

2. Model setup

2.1. Competition model

In Stackelberg competition, there are leading and follower enterprises. The leading enterprise first decides its own output according to profit maximization, and the following enterprise chooses its optimal output level based on the output of the leading enterprise. The difference between the Stackelberg and Cournot competition is that the former is a dynamic model, that is, the output decision is made in turn, while the latter is a static model, that is, the output decisions of all oligarchs are made simultaneously. We consider an oligopoly industry chain with one enterprise upstream and two enterprises downstream, named Enterprises 1 and 2, respectively. The three enterprises are either private or mixed-ownership enterprises. Three scenarios are discussed: an upstream mixed-ownership enterprise and two downstream private enterprises (Model MPP); an upstream private enterprise and two downstream mixed-ownership enterprises (Model PMM); and an upstream private enterprise, a downstream mixed-ownership enterprise, and a downstream private enterprise (Model PMP). With the exception of the third scenario, which competes in the Stackelberg competition with mixed-ownership enterprises as the leader, the others are in the Cournot competition.

2.2. Model assumptions

The assumptions of this study are summarized as follows.

Assumption 1. The price of the upstream enterprise raw material is p, and the prices of downstream enterprise products are $p_i = a - q_i - rq_j$, $(i, j = 1,2, \text{ and } i \neq j)$, $0 \leq r \leq 1$. r represents the level of product homogeneity. The higher r, the more homogeneous the products.

Assumption 2. The cost function of the upstream enterprise is $c = m\beta + n(1-\beta)$ and the cost function of the downstream company is $c_i = f\beta + e(1-\beta) + p$ (Chen et al., 2019c), where m and f are the costs of state-owned shares. In addition, n and e are the costs of private shares. Further, suppose $m > n$ and $f > e$, then private shares are more efficient. Moreover, the upstream product is the raw material of the downstream product; a unit of raw material upstream has a unit of output downstream. Thus, the price of the downstream product constitutes a part of the downstream cost. β is the proportion of state-owned shares and $0 \le \beta \le 1$. When $\beta = 0$, the upstream enterprise is private; when $\beta = 1$, the upstream enterprise is fully nationalized.

Assumption 3. The profit function of the upstream enterprise is $\pi = (p-c)(q_1+q_2)$, and its utility function is $U = \beta SW + (1-\beta)\pi$, where $SW = \pi + \pi_1 + \pi_2 + CS$ represents the social welfare of the industry chain. CS represents consumer surplus and $CS = \frac{q_1^2 + q_2^2 + 2rq_1q_2}{2}$ (Singh & Vives, 1984). The utility function of downstream enterprises are $U_i = \beta SW + (1-\beta)\pi_i$, where $\pi_i = (p_i - c_i)q_i$, (i, j = 1,2 and $i \neq j$).

Under the above assumptions, a two-stage game is adopted, in which the upstream enterprise decides the whole price (p). In stage two, downstream enterprises determine production output (q_1 and q_2). After obtaining the equilibrium results in the four scenarios, we analyze the impacts of β and r on them and perform a comparative analysis. In addition, this study also includes a scenario in which the government determines the optimal β based on the maximization of social welfare in the expansion part, expanding the game into three stages, and exploring the impacts of various factors.

3. Model analysis

3.1. Model MPP (an upstream mixed-ownership enterprise and two downstream private enterprises)

In the Model MPP, there is a mixed-ownership upstream enterprise and two private downstream enterprises.

The first step is to examine the second stage of the game. According to the profit maximization principle, downstream enterprises' outputs satisfy $\frac{\partial \pi_i}{\partial q_i} = 0$. They are obtained as

$$q_i = \frac{a-e-p}{2+r}. \tag{1}$$

Subsequently, by substituting Eq. (1) into the upstream enterprise utility function and solving for $\frac{\partial U}{\partial p} = 0$, the equilibrium price p^{MPP} is obtained as follows:

$$p^{MPP} = \frac{\left[(a-e)(3+r)-(m-n)(2+r)\right]\beta-(a-e+n)(2+r)}{\beta(3+r)-2(2+r)}. \tag{2}$$

Owing to $p^{MPP} \ge 0$ and $\pi^{MPP} \ge 0$, we can get $0 < \beta \le \frac{r+2}{r+3}$.

Analyzing the impacts of β and r on the equilibrium results, we can obtain Proposition 1:

Proposition 1:

(1) The effects of β:

$$\frac{\partial p^{MPP}}{\partial \beta}<0,\ \frac{\partial \pi^{MPP}}{\partial \beta}<0,\ \frac{\partial q_i^{MPP}}{\partial \beta}>0,\ \frac{\partial p_i^{MPP}}{\partial \beta}<0,\ \frac{\partial \pi_i^{MPP}}{\partial \beta}>0,\ \frac{\partial CS^{MPP}}{\partial \beta}>0.$$

If $0<\beta\le \dfrac{\begin{array}{c}(a-e-6m+5n)r+3(a-e-4m+3n)-\\ \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}\end{array}}{2(3+r)(m-n)}$,

then $\dfrac{\partial SW^{MPP}}{\partial \beta}\ge 0$.

If $\dfrac{\begin{array}{c}(a-e-6m+5n)r+3(a-e-4m+3n)-\\ \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}\end{array}}{2(3+r)(m-n)}<\beta<\dfrac{r+2}{r+3}$,

then $\dfrac{\partial SW^{MPP}}{\partial \beta}<0$.

(2) The effects of r:

$$\frac{\partial p^{MPP}}{\partial r}<0,\ \frac{\partial \pi^{MPP}}{\partial r}<0,\ \frac{\partial q_i^{MPP}}{\partial r}<0,\ \frac{\partial p_i^{MPP}}{\partial r}<0,\ \frac{\partial \pi_i^{MPP}}{\partial r}<0,\ \frac{\partial CS^{MPP}}{\partial r}<0,$$

$$\frac{\partial SW^{MPP}}{\partial r}<0.$$

Proof: See Appendix, A.

Note that proportion of state-owned shares is negatively correlated with the product prices of upstream and downstream enterprises and upstream mixed-ownership enterprises' profit, while it is positively correlated with the output, downstream private enterprises' profits, and consumer surplus. The proportion of state-owned shares has both positive and negative impacts on social welfare. These results show that increasing the proportion of state-owned shares reduces raw material prices and increases share costs, leading to a decline in profits.

Moreover, outputs increase as the proportion of state-owned shares increases, which exacerbates upstream enterprises' profit loss. Therefore, when the upstream enterprise is a mixed-ownership enterprise, considering its own profit, it has no motivation to increase its state-owned share proportion. The increase in the proportion of state-owned shares benefits downstream enterprises and consumers. For downstream enterprises, although the price of their products will decrease when the balance of state-owned shares increases, the reduction in costs compensates for the loss in their profits, ultimately leading to higher profits. For consumers, increasing state-owned shares will increase output, thereby increasing the consumer surplus. This finding suggests that if mixed ownership is implemented upstream, the upstream enterprises will increase the proportion of private shares, which will reduce downstream enterprises' profits and consumer surplus. Social welfare is enhanced only if the enterprise increases based on a low proportion of state-owned shares. Therefore, the balance of state-owned shares has an "inverted U-shaped" impact. Thus, an optimal ratio of state-owned shares exists. Considering social welfare, the government does not always support the behavior of increasing the proportion of state-owned shares.

The degree of product homogeneity also affects the equilibrium results. The higher the degree of product homogeneity, the lower the equilibrium results. When the degree of product homogeneity increases, prices decrease due to weakened social welfare. The outputs decrease, resulting in a decline in profits and consumer surplus, further reducing social welfare. Thus, enterprises are motivated to differentiate their products, and consumers and society benefit from these activities (Chen et al., 2021).

3.2. Model PMM (an upstream private enterprise and two downstream mixed-ownership enterprises)

In Model PMM, there is one private enterprise upstream and two mixed-ownership enterprises downstream. First, to maximize their utilities, the outputs satisfy $\frac{\partial U_i}{\partial q_i}=0$. The outputs can be expressed as

$$q_i=\frac{\beta\left(e-f-n+p\right)+a-e-p}{2+r-\beta}. \tag{3}$$

Subsequently, by substituting Eq. (3) into the upstream enterprise's profit function, according to profit maximization $\frac{\partial \pi}{\partial p}=0$, we can derive

$$p^{PMM}=\frac{\beta\left(e-f-2n\right)+a-e+n}{2\left(1-\beta\right)}. \tag{4}$$

Owing to $\pi_i^{PMM}\geq 0$, we can obtain $0<\beta\leq\frac{4+r-\sqrt{8+8r+r^2}}{4}$.

Proposition 2:

(1) The effects of β:

$$\frac{\partial p^{PMM}}{\partial \beta}>0,\ \frac{\partial \pi^{PMM}}{\partial \beta}>0,\ \frac{\partial q_i^{PMM}}{\partial \beta}>0,\ \frac{\partial p_i^{PMM}}{\partial \beta}<0,\ \frac{\partial \pi_i^{PMM}}{\partial \beta}<0,\ \frac{\partial CS^{PMM}}{\partial \beta}>0,$$
$$\frac{\partial SW^{PMM}}{\partial \beta}>0.$$

(2) The effects of r:

$$\frac{\partial p^{PMM}}{\partial r}<0,\ \frac{\partial \pi^{PMM}}{\partial r}<0,\ \frac{\partial q_i^{PMM}}{\partial r}<0,\ \frac{\partial p_i^{PMM}}{\partial r}<0,\ \frac{\partial \pi_i^{PMM}}{\partial r}<0,\ \frac{\partial CS^{PMM}}{\partial r}<0,$$
$$\frac{\partial SW^{PMM}}{\partial r}<0.$$

Proof: See Appendix, B.

Compared with the Model MPP, the proportion of state-owned shares in upstream and downstream enterprises and social welfare changes due to the different links in the industry chain implementing mixed ownership. Upstream enterprises' raw material prices and profits have changed from a negative to a positive correlation with the proportion of state-owned shares. The earnings of downstream enterprises have changed from a positive to a negative correlation. Social welfare is always positively correlated with the proportion of state-owned shares. Upstream, increasing the proportion of state-owned shares will increase the price, while the cost will not change, leading to higher profits. However, lower prices and higher costs will result in lower downstream profits. In addition, two downstream mixed-ownership enterprises will increase their outputs as the proportion of their own state-owned shares increases, exacerbating their losses. If two downstream enterprises implement mixed ownership, a higher state-owned share ratio is unsuitable. By contrast, a higher state-owned share ratio benefits upstream enterprises, consumers, and society, resulting in a conflict analogous to Model MPP. The downstream enterprise pursuing the maximization of profits will reduce the proportion of state-owned shares, even if there are no state-owned shares, and harm stakeholders' interests.

Similarly, product differentiation is still negatively correlated with each equilibrium result; however, the degree of correlation is different. Since product differentiation increases profits, three enterprises in the industry chain are motivated to conduct product innovation (Chu et al., 2019; Tan et al., 2020). They can develop horizontal and vertical cooperation between enterprises to achieve a win-win situation.

3.3. Model PMP (an upstream private enterprise, a downstream mixed-ownership enterprise, and a downstream private enterprise)

In Model PMP, there is a private enterprise upstream and mixed-ownership Enterprise 1 and private Enterprise 2 downstream. The mixed-ownership enterprise has a comparative advantage in competition; thus, the Stackelberg dynamic competition model is adopted with Enterprise 1 as the leader and Enterprise 2 as the follower. First, Enterprise 2 decides to maximize profit. The equilibrium output is derived as follows:

$$q_2 = \frac{a - e - p - rq_1}{2}. \tag{5}$$

Subsequently, by substituting Eq. (6) into the utility function of Enterprise 2, and obtaining the equilibrium output of Enterprise 1 according to utility maximization, we obtain the equilibrium outputs as

$$q_1 = \frac{\left[\left(a - e + p - 2n\right)r - 4\left(e - f + p - n\right)\right]\beta - 2\left(a - e - p\right)\left(4 - 2r\right)}{\left[\left(4 - r^2\right)\beta - 4\left(2 - r^2\right)\right]}, \tag{6}$$

$$q_2 = -\frac{\left[\left(a - e - n\right)r^2 - 2\left(e - f - n + p\right)r - 2\left(a - e - p\right)\right]\beta - \left(a - e - p\right)\left(r^2 + 2r - 4\right)}{\left[\left(4 - r^2\right)\beta - 4\left(2 - r^2\right)\right]}. \tag{7}$$

Finally, by using Eqs (6) and (7) to maximize the profit of the upstream enterprise, the equilibrium price is derived as

$$p^{PMP} = -\frac{\left[\left(a - e - n\right)r^2 - \left(a + e - 2f - 7n\right)r - 2\left(a - 3e + 2f + 5n\right)\right]\beta - \left(a - e + n\right)\left(r^2 + 4r - 8\right)}{2\left[3\left(2 - r\right)\beta - \left(8 - 4r - r^2\right)\right]}. \tag{8}$$

Proposition 3:

(1) The effects of β:

$$\frac{\partial p^{PMP}}{\partial \beta} > 0,\ \frac{\partial \pi^{PMP}}{\partial \beta} > 0,\ \frac{\partial q_1^{PMP}}{\partial \beta} > 0,\ \frac{\partial q_2^{PMP}}{\partial \beta} < 0,\ \frac{\partial p_1^{PMP}}{\partial \beta} < 0,\ \frac{\partial p_2^{PMP}}{\partial \beta} > 0,\ \frac{\partial \pi_1^{PMP}}{\partial \beta} < 0,$$
$$\frac{\partial \pi_2^{PMP}}{\partial \beta} < 0,\ \frac{\partial CS^{PMP}}{\partial \beta} > 0,\ \frac{\partial SW^{PMP}}{\partial \beta} > 0.$$

(2) The effects of r:

$$\frac{\partial \pi^{PMP}}{\partial r} < 0,\ \frac{\partial q_1^{PMP}}{\partial r} < 0,\ \frac{\partial q_2^{PMP}}{\partial r} < 0,\ \frac{\partial p_1^{PMP}}{\partial r} < 0,\ \frac{\partial p_2^{PMP}}{\partial r} < 0,\ \frac{\partial \pi_1^{PMP}}{\partial r} < 0,\ \frac{\partial \pi_2^{PMP}}{\partial r} < 0,$$
$$\frac{\partial CS^{PMP}}{\partial r} < 0,\ \frac{\partial SW^{PMP}}{\partial r} < 0. \text{ If } 0 < r \le r_1, \text{ then } \frac{\partial p^{PMP}}{\partial r} \le 0, \text{ if } r_1 < r < 1, \text{ then } \frac{\partial p^{PMP}}{\partial r} > 0.$$

Proof: See Appendix, C.

The impacts of the proportion of state-owned shares on the equilibrium result differ from those in the Model MPP and Model PMM. As the proportion of state-owned shares increases, raw material prices and upstream enterprises' profits, consumer surplus, and social welfare increase, while costs and profits decrease downstream. The proportion of state-owned shares is negatively related to the earnings of the two downstream enterprises, but for different reasons. If the balance of state-owned shares increases, lower product prices, higher share costs, and higher outputs of enterprises implementing mixed ownership lead to lower profits. However, when the proportion of competitors' state-owned shares increases, although their product prices increase, the profits of enterprises that do not implement mixed ownership decline due to the increase in upstream product prices and the decrease in their outputs.

Compared with Model MPP, the proportion of state-owned shares has the opposite effect on the prices of upstream raw materials and profits, downstream enterprises that implement mixed ownership, and the output and earnings of downstream private enterprises. Compared with Model PMM, only the impacts on production and price are the opposite. Changes in the effects on upstream and downstream enterprises that implement mixed ownership are caused by converting the implementation link from upstream to downstream. The implementation of mixed ownership is thus induced. Unlike the implementation of mixed ownership upstream of the industry chain and in the two downstream enterprises, the relationship between private enterprises and the proportion of state-owned shares has changed. When only one enterprise downstream is a mixed-ownership enterprise, the increase in the proportion of state-owned shares will encroach on the output of competitors, leading to the disadvantage of lower profits despite the increase in the prices of their products.

The impact of product differentiation on upstream raw material prices has changed. The cost of raw materials and degree of product differentiation are first positively and then negatively correlated. The other equilibrium results are negatively correlated with product differentiation. Chen et al. (2020) have argued that product differentiation affects output, profits, consumer surplus, and social welfare, and that these effects are constrained by capacity constraints and capacity sharing. In contrast, we investigate product differentiation in the industry chain. When the degree of homogeneity of downstream products is high, the price of upstream products no longer increases due to a decrease in the degree of differentiation of downstream products. Although the impact on prices changes when the level of product differentiation is higher, the upstream enterprise will ultimately support downstream enterprises to improve their product differentiation for profit maximization.

3.4. Model PPP (an upstream private enterprise and two downstream private enterprises)

In Model PPP, there are three private enterprises. Two downstream enterprises make decisions based on profit maximization and satisfy $\frac{\partial \pi_i}{\partial q_i} = 0$. The equilibrium outputs are obtained as follows:

$$q_i = \frac{a-e-p}{2+r}. \tag{9}$$

Second, by substituting Eq. (9) into the utility function of the upstream enterprise, the equilibrium price according to profit maximization can be derived as

$$p^{PPP} = \frac{a-e+n}{2}. \tag{10}$$

Proposition 4:

The effects of r:

$$\frac{\partial p^{PPP}}{\partial r} = 0,\ \frac{\partial \pi^{PPP}}{\partial r} < 0,\ \frac{\partial q_i^{PPP}}{\partial r} < 0,\ \frac{\partial p_i^{PPP}}{\partial r} < 0,\ \frac{\partial \pi_i^{PPP}}{\partial r} < 0,\ \frac{\partial CS^{PPP}}{\partial r} < 0,\ \frac{\partial SW^{PPP}}{\partial r} < 0.$$

Proof: See Appendix, D.

The outcomes in Proposition 4 imply that the equilibrium results are all negatively correlated, except that the upstream product price has nothing to do with the degree of product differentiation when all three enterprises are private. Further, regardless of whether the enterprise implements mixed ownership, product differentiation is crucial for improving profits, consumer surplus, and social welfare.

4. Discussion section

4.1. Comparison

Corollary 1 can be obtained by comparing the equilibrium results in the four scenarios. To ensure that the results are positive, $0 < \beta \le \frac{4+r-\sqrt{8+8r+r^2}}{4}$.

Corollary 1:

$p^{PMM} > p^{PPP} > p^{PMP} > p^{MPP}$, $\pi^{PMM} > \pi^{PMP} > \pi^{MPP}$, if $a_0 < a \le a_1$, then $\pi^{PMP} \le \pi^{PPP}$; if $a > a_1$, then $\pi^{PMP} > \pi^{PPP}$; $q_1^{MPP} > q_1^{PMM} > q_1^{PPP}$, if $a_0 < a \le a_2$, then $q_1^{PMP} \le q_1^{MPP}$; if $a > a_2$, then $q_1^{PMP} > q_1^{MPP}$, $q_2^{MPP} > q_2^{PMM} > q_2^{PPP} > q_2^{PMP}$; $p_1^{PPP} > p_1^{PMM} > p_1^{MPP}$, if $a_0 < a \le a_3$, then $p_1^{MPP} \le p_1^{PMP}$; if $a > a_3$, then $p_1^{MPP} > p_1^{PMP}$, $p_2^{PMP} > p_2^{PPP} > p_2^{PMM} > p_2^{MPP}$; $\pi_1^{MPP} > \pi_1^{PPP} > \pi_1^{PMP} > \pi_1^{PMM}$,

$\pi_2^{MPP} > \pi_2^{PPP} > \pi_2^{PMP} > \pi_2^{PMM}$, $CS^{MPP} > CS^{PMP} > CS^{PMM} > CS^{PPP}$, $SW^{MPP} > SW^{PMM} > SW^{PMP}$, if $a_0 < a \leq a_4$, then $SW^{PMM} \leq SW^{PPP}$; if $a > a_4$, then $SW^{PMM} > SW^{PPP}$.

Proof: See Appendix, E.

Table 1. Comparing results of the MPP, PMM, and PMP models

Item	Comparing results	Co-evaluation
Upstream price	$p^{PMM} > p^{PMP} > p^{MPP}$	All in the same order
Upstream profit	$\pi^{PMM} > \pi^{PMP} > \pi^{MPP}$	
Downstream outputs	$q_1^{MPP} > q_1^{PMM}$ if $a_0 < a \leq a_2$, then $q_1^{PMP} \leq q_1^{MPP}$, if $a > a_2$, then $q_1^{PMP} > q_1^{MPP}$	MPP and PMM in the same order
	$q_2^{MPP} > q_2^{PMM} > q_2^{PMP}$	
Downstream prices	$p_1^{PMM} > p_1^{MPP}$ if $a_0 < a \leq a_3$, then $p_1^{MPP} \leq p_1^{PMP}$, if $a > a_3$, then $p_1^{MPP} > p_1^{PMP}$	MPP and PMM in the same order
	$p_2^{PMP} > p_2^{PMM} > p_2^{MPP}$	
Downstream profits	$\pi_1^{MPP} > \pi_1^{PMP} > \pi_1^{PMM}$	All in the same order
	$\pi_2^{MPP} > \pi_2^{PMP} > \pi_2^{PMM}$	
Consumer surplus	$CS^{MPP} > CS^{PMP} > CS^{PMM}$	
Social welfare	$SW^{MPP} > SW^{PMM} > SW^{PMP}$	In the same order with Enterprise 2's output

Table 1 presents the comparing results of the MPP, PMM, and PMP models. By comparison, it can be seen that when upstream mixed ownership is implemented, the upstream raw material price is the lowest. The prices of downstream enterprises are slightly higher when a mixed-ownership system is implemented; the price is higher when all three enterprises are private. The raw material price is highest when the two downstream enterprises implement mixed ownership. For the upstream enterprise's profits, the maximum value is achieved when the two downstream enterprises implement mixed ownership; the lowest is when the upstream enterprise implements mixed ownership, and the middle is when the downstream enterprise implements mixed ownership. Third, all enterprises are private enterprises. The relationship between them is related to the potential downstream market size. When the downstream market is significant and a downstream enterprise implements mixed-ownership reform, the upstream enterprise has higher profits.

Conversely, the opposite result is obtained when it is small. The outputs of the two downstream enterprises are similar, but the difference lies in the upstream enterprise's implementation of mixed ownership and its performance. When mixed ownership exists, the output is uncertain. When the potential scale of the downstream market is large, the output will be higher when downstream enterprises implement mixed ownership. The price relationship between the two downstream enterprises is also similar. The difference lies in comparing the links that implement mix ownership; however, the price is higher when the potential market is small.

Downstream profits are highest when they implement mixed ownership, followed by Model PPP, then Model PMP, and lowest in the Model PMM. Simultaneously, consumer surplus and social welfare are the largest in the Model MPP, slightly smaller in the Model PMM, and the smallest in the case of Model PMP. From the viewpoint of social welfare, the relationship between Model PMM and Model PPP is also related to the potential downstream market size. When the downstream market is large, social welfare is higher if downstream enterprises are of mixed ownership. From the industry chain perspective, when upstream enterprises implement mixed ownership, their product prices and profits are the lowest. Thus, upstream enterprises may be unwilling to promote mixed-ownership reform, which requires government support and encouragement.

In contrast, downstream enterprises have the highest output, profits, and consumer surplus. This implies that when upstream enterprises implement mixed ownership, the impact of mixed ownership on the enterprise will be transmitted downstream through the industry chain, for example, by reducing the price of raw materials. Therefore, the policy effect of implementing mixed ownership upstream is more prominent. As far as the government is concerned, it can prioritize implementing mixed ownership upstream and make full use of the essential role of the upstream. Moreover, implementing mixed ownership downstream requires focusing on the critical factor of downstream market size.

4.2. Expansion

It is essential to determine the optimal proportion of state-owned shares in an industry chain. The game is expanded in three stages. In stage one, the government determines the optimal balance of state-owned shares (β) according to social welfare maximization. In stage two, the upstream price determines the total price (p). In stage three, downstream enterprises compete for outputs (q_1 and q_2). Based on the previous analysis, assume β is endogenous and incorporate the government into the game in this part (Chen et al., 2009).

Corollary 2: The optimal proportion of state-owned shares in the three situations is

$$\beta^{MPP} = \frac{\begin{array}{c}(a-e-6m+5n)r+3(a-e-4m+3n)- \\ \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}\end{array}}{2(3+r)(m-n)};$$

$$\beta^{PMM} = \frac{4+r-\sqrt{8+8r+r^2}}{4};\ \beta^{PMP} = 1,$$

where $F = \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}$.

Next, we explore various factors that impact the proportion of state-owned shares. Corollary 3 can be derived as follows:

Corollary 3:

$$\frac{\partial\beta^{MPP}}{\partial m} < 0;\ \frac{\partial\beta^{MPP}}{\partial n} > 0;\ \frac{\partial\beta^{MPP}}{\partial e} < 0;\ \frac{\partial\beta^{MPP}}{\partial r} < 0;\ \frac{\partial\beta^{PMM}}{\partial r} < 0.$$

Proof: See Appendix, F.

In the Model MPP, the optimal proportion of state-owned shares is β^{MPP}. It is affected by share costs and the degree of product differentiation. State-owned share costs upstream and private share costs downstream are negatively correlated. Upstream private share costs and product differentiation are positively correlated. These results imply that when state-owned share costs are lower or the degree of product differentiation is higher, the optimal proportion of state-owned shares will be higher. Bárcena-Ruiz and Garzón (2018) examined privatization in terms of differentiation and bargaining power. Cho et al. (2022) investigated the optimal degree of privatization by considering open technologies and fee-based licensing strategies. We consider state-owned shares and product differentiation to determine the optimal level of social welfare. From the perspective of private share costs, the higher the cost of upstream private shares or the lower the cost of downstream private shares, the higher the optimal proportion of state-owned shares. In the case of Model PMM, the optimal balance of state-owned shares is β^{PMM}, which increases only if the degree of product differentiation is higher.

Moreover, downstream enterprises are squeezed out, thereby realizing the monopoly of upstream enterprises. In the case of Model PMP, the optimal proportion of state-owned shares is $\beta^{PMP} = 1$. This is independent of other factors. Therefore, to determine the degree of nationalization when implementing mixed ownership, it is necessary to conduct a comprehensive analysis of industry chain links, share costs, and the degree of product differentiation. When the implementation link is upstream and the cost of state-owned shares is lower, it is more beneficial to society if the proportion of state-owned shares is higher. However, share costs no longer need to be considered when the implementation link is downstream. Complete nationalization is the best choice, only if a downstream enterprise implements mixed ownership.

4.3. Practical significance

Taking China as an example, mixed-ownership reform and industrial chain optimization in many industries are closely integrated and synergistic, and the fact has proved that mixed ownership has become an effective way to modernize the industrial chain. Thus, it is necessary to improve mixed-ownership reform based on the overall layout of the industrial chain. In the process of promoting sustainable industrial development, it is necessary to pay attention to the behavior of stakeholders and industrial environment. When conducting production evaluation and strategic selection, both the performance of individual enterprises and the industry as a whole should be considered to evaluate the performance of industrial organizations from an environmental perspective (Ahmed et al., 2021; Alfina & Ratnayake, 2019; Aravossis et al., 2019). These should be the important goals and directions of mixed-ownership reform in the industrial chain.

Conclusions

This study constructs an oligopoly competition model in the industry chain, including mixed ownership and private enterprises, investigates the impact of the proportion of state-owned shares and product differentiation on the equilibrium results, and expands the discussion by comparing the equilibrium results under different situations and determining the optimal proportion of state-owned shares. The following conclusions are drawn.

The government should adopt the necessary mechanism design while implementing mixed ownership, such as by setting the lower limit of the proportion of state-owned shares. First, the proportion of state-owned shares affects profits, consumer surplus, and social welfare. As the proportion of state-owned shares increases, price decreases, but the profits of private enterprises and consumer surplus increase. This is because when enterprises implement mixed ownership and increase the proportion of state-owned shares, other enterprises' prices will decrease less than raw material or share costs. Thus, the government should encourage qualified enterprises to implement mixed-ownership reforms and continuously optimize the proportion of state-owned shares. Moreover, enterprises implementing mixed ownership are preferable for increasing their privatization to increase profits, causing damage to private enterprises and consumers.

Second, product differentiation has a significant influence on equilibrium results. Positive relationships between product differentiation and profits, consumer surplus, and social welfare always exist in all four cases. This is because prices and outputs increase when the degree of product differentiation is high. In contrast to Model PMP, upstream prices will increase if the degree of product differentiation is low. In general, product differentiation is always beneficial to enterprises, consumers, and society. Therefore, the government should encourage enterprises to engage in product innovation, while implementing mixed ownership.

Third, industry chain links and market scales that implement mixed ownership differ, inducing different equilibrium results. When mixed ownership is implemented upstream, the price of raw materials is the lowest. However, the largest are downstream enterprises' profits and outputs, consumer surplus, and social welfare. Therefore, for the government, upstream enterprises should prioritize the implementation of mixed ownership. The policy effect is more evident here. Moreover, the comparison results are affected by potential downstream market size. Implementing mixed ownership in downstream enterprises is feasible when the downstream market is large. If downstream enterprises have implemented mixed ownership, the government should continue to encourage competitors to implement mixed ownership to further improve the policy effect. However, negative impacts such as upstream monopolies should be avoided.

Fourth, the optimal proportion of state-owned shares to maximize the social welfare of the industry chain should be chosen by considering many relative factors. The optimal balance of state-owned shares in the Model MPP is affected by share costs and the degree of product differentiation. In the Model PMM and Model PMP, the optimal proportion of state-owned shares is obtained at the boundary. In the Model PMM, the optimal balance of state-owned shares is affected only by the degree of product differentiation. By contrast, complete nationalization becomes the optimal choice in the Model PMP. These results suggest that the government must fully consider factors such as industry chain links, share costs, and product differentiation when determining the proportion of state-owned shares. If the implementation link is upstream, the balance of state-owned shares should be adjusted to close to but not exceed the optimal state-owned share ratio. Otherwise, it will harm society. While the implementation link is downstream, the balance of state-owned shares should be increased as much as possible, and even fully nationalized.

This study has some limitations, which also provide directions for future research. First, it only considers the case of one upstream enterprise and two downstream enterprises. With rapid economic development, the relationship and quantity of upstream and downstream are becoming more complicated, and a more realistic model design is required. Second, this study only focuses on state-owned and private shares, but ignores other forms. Given the diversification of equity forms, it is important to explore the effects of other shares in the future.

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Author contributions

Xiao Kong and Junlong Chen conceived the study and contributed to the design of model. Jiali Liu was responsible for the topic selection of the paper. Xiao Kong and Chaoqun Sun were responsible for the derivation and analysis of the model. Xiao Kong and Chaoqun Sun wrote the first draft of the article. Jiali Liu and Junlong Chen checked and revised the draft.

Disclosure statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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APPENDIX

A: Proof of Proposition 1

(1) The effects of β:

$$\frac{\partial p^{MPP}}{\partial \beta} = -\frac{(2+r)\left[(a-e-2m+n)r+3a-3e-4m+n\right]}{[(3+r)\beta-2(2+r)]^2} < 0;$$

$$\frac{\partial q_i^{MPP}}{\partial \beta} = \frac{(a-e-2m+n)r+3a-3e-4m+n}{[(3+r)\beta-2(2+r)]^2} > 0;$$

$$\frac{\partial p_i^{MPP}}{\partial \beta} = -\frac{(r+1)\left[(a-e-2m+n)r+3a-3e-4m+n\right]}{[(3+r)\beta-2(2+r)]^2} < 0;$$

$$\frac{\partial \pi^{MPP}}{\partial \beta} = \frac{\begin{array}{c}2\left[a-e-n-(m-n)\beta\right]\{(m-n)(3+r)^2\beta^2 - \\ \left[(a-e-6m+5n)r+3(a-e-4m+3n)\right](3+r)\beta+4(m-n)(2+r)^2\}\end{array}}{-\left[(3+r)\beta\right]-2(2+r)]^3} < 0;$$

$$\frac{\partial \pi_i^{MPP}}{\partial \beta} = -\frac{2\left[a-e-n-(m-n)\beta\right]\left[(a-e-2m+n)r+3a-3e-4m+n\right]}{[(3+r)\beta-2(2-r)]^3} > 0;$$

$$\frac{\partial CS^{MPP}}{\partial \beta} = -\frac{2(1+r)\left[a-e-n-(m-n)\beta\right]\left[(a-e-2m+n)r+3a-3e-4m+n\right]}{[(3+r)\beta-2(2+r)]^3} > 0;$$

$$\frac{\partial SW^{MPP}}{\partial \beta} = \frac{\begin{array}{c}2\{(3+r)^2(m-n)^2\beta^3-6(3+r)(2+r)(m-n)^2\beta^2-\{(a-e-6m+5n)(a-e+m-2n)r^2+[6(a-e)^2- \\ 24(a-e)m+12(a-e)n-26m^2+76mn-44n^2]r+9(a-e)^2-27(a-e)m+9(a-e)n-28m^2+83mn \\ -46n^2\}\beta+(a-e-n)\left[(a-e-6m+5n)r^2+2(3a-3e-13m+10n)r+(9a-9e-28m+19n)\right]\}\end{array}}{-[(3+r)\beta-2(2+r)]^3}.$$

If $0<\beta\le\dfrac{\begin{array}{c}(a-e-6m+5n)r+3(a-e-4m+3n)- \\ \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}\end{array}}{2(3+r)(m-n)}$, then

$\dfrac{\partial SW^{MPP}}{\partial \beta}\ge 0$.

If $\dfrac{\begin{array}{c}(a-e-6m+5n)r+3(a-e-4m+3n)- \\ \sqrt{\left[(a-e-2m+n)r+3a-3e-4m+n\right]\left[(a-e-6m+5n)r+3a-3e-8m+5n\right]}\end{array}}{2(3+r)(m-n)}<\beta<\dfrac{r+2}{r+3}$, then

$\dfrac{\partial SW^{MPP}}{\partial \beta}<0$.

(2) The effects of r:

$$\frac{\partial p^{MPP}}{\partial r}=\frac{\beta\left[(a-e-n)-(m-n)\beta\right]}{[(3+r)\beta-2(2+r)]^2}<0\,;\ \frac{\partial q_i^{MPP}}{\partial r}=-\frac{(2-\beta)\left[(a-e-n)-(m-n)\beta\right]}{[(3+r)\beta-2(2+r)]^2}<0\,;$$

$$\frac{\partial p_i^{MPP}}{\partial r}=-\frac{2(1-\beta)\left[(a-e-n)-(m-n)\beta\right]}{[(3+r)\beta-2(2+r)]^2}<0\,;$$

$$\frac{\partial \pi^{MPP}}{\partial r}=\frac{2[(a-e-n-(m-n)\beta\big]^2\left[(3+r)\beta^2-3(3+r)\beta+2(2+r)\right]}{[(3+r)\beta-2(2+r)]^3}<0\,;$$

$$\frac{\partial \pi_i^{MPP}}{\partial r}=\frac{2[a-e-n-(m-n)\beta]^2\,(2-\beta)}{[(3+r)\beta-2(2+r)]^3}<0\,;$$

$$\frac{\partial CS^{MPP}}{\partial r}=\frac{[a-e-n-(m-n)\beta]^2\left[2r+(1-r)\beta\right]}{[(3+r)\beta-2(2+r)]^3}<0\,;$$

$$\frac{\partial SW^{MPP}}{\partial r}=\frac{\big[a-e-n-(m-n)\beta]^2\big[2(3+r)\beta^2-7(3+r)\beta+(3r+8)]}{[(3+r)\beta-2(2+r)]^3}<0\,.$$

B: Proof of Proposition 2

(1) The effects of β:

$$\frac{\partial p^{PMM}}{\partial \beta}=\frac{a-f-n}{2(\beta-1)^2}>0\,;\ \frac{\partial q_i^{PMM}}{\partial \beta}=\frac{(e-f)r+a+e-2f-n}{(\beta-2-r)^2}>0\,;$$

$$\frac{\partial p_i^{PMM}}{\partial \beta}=-\frac{(r+1)\left[(e-f)r+a+e-2f-n\right]}{2(-\beta+2+r)^2}<0\,;$$

$$\frac{\partial \pi^{PMM}}{\partial \beta}=-\frac{\left[a-e-n+(e-f)\beta\right]\left\{\left[2a+e-3f-2n+(e-f)r\right]\beta-\left[(a+e-2f-n)r+3a+e-4f-3n\right]\right\}}{2(1-\beta)^2(2+r-\beta)^2}>0;$$

$$\frac{\partial \pi_i^{PMM}}{\partial \beta}=\frac{\begin{array}{c}\left[a-e-n+(e-f)\beta\right]\{2(e-f)\beta^4-\left[6(e-f)r+2a+14e-16f-2n\right]\beta^3+[2(e-f)r^2+\\ 21(e-f)r+4a+31e-35f-4n\big]\beta^2-\big[3(e-f)r^2-(3a-22e+19f-3n)r-\\ (a-27e+26f-n)\big]\beta-\big[(a-e-n)r^2+(5a-7e+2f-5n)r+4(a-2e+f-n)\}\end{array}}{4(1-\beta)^2(2+r-\beta)^3}<0;$$

$$\frac{\partial CS^{PMM}}{\partial \beta}=\frac{(1+r)\left[a-e-n+(e-f)\beta\right]\left[a+e-2f-n+(e-f)r\right]}{2(2+r-\beta)^2}>0\,;$$

$$\frac{\partial SW^{PMM}}{\partial \beta}=\frac{\begin{array}{c}\left[a-e-n+(e-f)\beta\right]\{2(e-f)\beta^2-[6(e-f)r+2(a+5e-6f-n)\big]\beta+\big[3(e-f)r^2\\ +(a+12e-13f-n)r+3a+11e-14f-3n]\}\end{array}}{2(2+r-\beta)^3}>0.$$

(2) The effects of r:

$$\frac{\partial p^{PMM}}{\partial r}=0\,;\ \frac{\partial q_i^{PMM}}{\partial r}=\frac{(f-e)\beta-a+e+n}{2(-\beta+2+r)^2}<0\,;\ \frac{\partial p_i^{PMM}}{\partial r}=\frac{(\beta-1)\left[(a-e-n)+(e-f)\beta\right]}{2(-\beta+2+r)^2}<0\,;$$

$$\frac{\partial \pi^{PMM}}{\partial r} = -\frac{[a-e-n+(e-f)\beta]^2}{2(2+r-\beta)^2(1-\beta)} < 0\,;$$

$$\frac{\partial \pi_i^{PMM}}{\partial r} = \frac{[a-e-n+(e-f)\beta]^2[3\beta^2-(6+r)\beta+2]}{-4(2+r-\beta)^3(1-\beta)} < 0\,;$$

$$\frac{\partial CS^{PMM}}{\partial r} = -\frac{(\beta+r)[a-e-n+(e-f)\beta]^2}{4(2+r-\beta)^3} < 0\,;$$

$$\frac{\partial SW^{PMM}}{\partial r} = \frac{[a-e-n+(e-f)\beta]\{5(e-f)\beta^2-[3(e-f)r-(5a-13e+8f-5n)]\beta-(a-e-n)(8+3r)\}}{4(2+r-\beta)^3} < 0.$$

C: Proof of Proposition 3

(1) The effects of β:

$$\frac{\partial p^{PMP}}{\partial \beta} = \frac{(2-r)(r^2+4r-8)[(a-e-n)r-2(a-f-n)]}{2[r^2+(4-3\beta)r+6\beta-8]^2} > 0\,;$$

$$\frac{\partial q_1^{PMP}}{\partial \beta} = \frac{\begin{gathered}\{(\beta^2-5\beta-2)(a-e-n)r^3+[(a-3e+2f-n)\beta^2+2(5a+e-6f-5n)\beta-4(a-e-n)]r^2+[-2(5a+e-6f-5n)\beta^2-\\ 8(a-3e+2f-n)\beta+32(a-e-n)]r+8(a+3e-4f-n)\beta^2+8(a-7e+6f-n)\beta-32(a-e-n)\}(2-r)[r^3-6(\beta-3)r^2+\\ 24\beta-40]+\{(a-e-n)(2\beta-5)r^3+[2(a-3e+2f-n)\beta+2(5a+e-6f-5n)]r^2+[-4(5a+e-6f-5n)\beta-8(a-\\ 3e+2f-n)]r16(a+3e-4f-n)\beta+8(a-7e+6f-n)\}[r^2+(-3\beta+4)r+6\beta-8][(\beta-4)r^2-4\beta+8]\end{gathered}}{2[r^2+(-3\beta+4)r+6\beta-8]^2[(\beta-4)r^2-4\beta+8]^2} > 0;$$

$$\frac{\partial q_2^{PMP}}{\partial \beta} = \frac{\begin{gathered}\{(\beta-1)(a-e-n)r^4+[-4(a-e-n)\beta^2+(11a-13e+2f-11n)\beta-6(a-e-n)]r^3+2(\beta-2)[4(a-f-n)\beta-\\ (a-e-n)]r^2[2(5a-11e+6f-5n)\beta^2-8(5a-7e+2f-5n)\beta+32(a-e-n)]r-4(\beta-2)[(5a-3e-2f-5n)\beta-\\ 4(a-e-n)]\}(2-r)[r^3-[4(5a-11e+6f-5n)\beta-8(5a-7e+2f-5n)]r-[8(5a-3e-2f-5n)\beta+8(7a-\\ 5e-2f-5n)\beta+8(7a-5e-2f-7n)]\}[r^2+(-3\beta+4)r+6\beta-8][(\beta-4)r^2-4\beta+8]\end{gathered}}{2[r^2+(-3\beta+4)r+6\beta-8]^2[(\beta-4)r^2-4\beta+8]^2} > 0;$$

$$\frac{\partial p_1^{PMP}}{\partial \beta} = \frac{\begin{gathered}\{-(\beta-1)(a-e-n)r^5+[4(a-e-n)\beta^2-(9a-13e+2f-11n)\beta-2(a+3e+3n)]r^4+[(-15a+e+8f+9n)\beta^2+\\ (55a-7e-16f-23n)\beta-2(17a-e-n)]r^3+[(a+25e-14f+11n)\beta^2-2(21a+29e-14f+15n)\beta+4(13a+\\ 7e+7n)]r^2+[2(27a-5e-10f-15n)\beta^2-16(8a-e-2f-3n)\beta+64a]r-8(7a+3e-4f-n)\beta^2+8(19a+\\ 7e-6f+n)\beta-32(3a+e+n)\}(2-r)[r^3-6(\beta-3)r^2+24\beta-40]+\{-(a-e-n)r^5+[8(a-e-n)\beta-(9a-13e+\\ 2f-11n)]r^4+[2(-15a+e+8f+9n)\beta+55a-7e-16f-23n]r^3+[2(a+25e-14f+11n)\beta-2(21a+29e-\\ 14f+15n)]r^2+[4(27a-5e-10f-15n)\beta-16(8a-e-2f-3n)]r-16(7a+3e-4f-n)\beta+8(19a+7e-\\ 6f+n)\}[(\beta-4)r^2-4\beta+][r^2+(-3\beta+4)4+6\beta-8]\end{gathered}}{2[(\beta-4)r^2-4\beta+8]^2[r^2+(-3\beta+4)r+6\beta-8]^2} > 0;$$

$$\frac{\partial p_2^{PMP}}{\partial \beta} = \frac{\begin{gathered}\{[2(a-e-n)\beta-2(3a-2e-2n)]r^4+[2(3a+e+2f+3n)\beta-11a-11e-10f-21n]r^3+[-4(7a+e-2f-n)\beta+\\ 2(23a+13e+13n)]r^2+[(-4(3a-e+10f+9n)\beta+16(3a+2f+2n)]r+8(7a+3e+2f+5n)\beta-8(13a+5e+2f+\\ 7n)\}[(\beta-4)r^2-4\beta+8][r^2+(-3\beta+4)r+6\beta-8]-\{[(a-e-n)\beta^2-2(3a-2e-2n)\beta+5a+3e+3n]r^4+[(3a+\\ e+2f+3n)\beta^2-(11a+11e+10f+21n)\beta+22a+10e+10n]r^3+[-2(7a+e-2f-n)\beta^2+2(23a+13e+13n)\beta-\\ 44a-36e-36n]r^2+[(-2(3a-e+10f+9n)\beta^2+16(3a+2f+2n)\beta-64a]r+4(\beta-2)[(7a+3e+2f+5n)\beta-\\ 4(3a+e+n)]\}(r+2)[r^3-2(3\beta-9)r^2+24\beta-40]\end{gathered}}{2[(\beta-4)r^2-4\beta+8]^2[r^2+(-3\beta+4)r+6\beta-8]^2} > 0;$$

$$\frac{\partial \pi^{PMP}}{\partial \beta} = \left(\frac{\partial p^{PMP}}{\partial \beta}-\frac{\partial c^{PMP}}{\partial \beta}\right)\left(q_1^{PMP}+q_2^{PMP}\right)+\left(p^{PMP}-c^{PMP}\right)\left(\frac{\partial q_1^{PMP}}{\partial \beta}+\frac{\partial q_2^{PMP}}{\partial \beta}\right) > 0,$$

$$\frac{\partial c^{PMP}}{\partial \beta} = m-n;$$

$$\frac{\partial \pi_1^{PMP}}{\partial \beta} = \left(\frac{\partial p_1^{PMP}}{\partial \beta} - \frac{\partial c_1^{PMP}}{\partial \beta} \right) q_1^{PMP} + \left(p_1^{PMP} - c_1^{PMP} \right) \frac{\partial q_1^{PMP}}{\partial \beta} < 0 \text{, where}$$

$$\frac{\partial c_1^{PMP}}{\partial \beta} = f - e + \frac{\partial p^{PMP}}{\partial \beta};$$

$$\frac{\partial \pi_2^{PMP}}{\partial \beta} = \left(\frac{\partial p_2^{PMP}}{\partial \beta} - \frac{\partial c_2^{PMP}}{\partial \beta} \right) q_2^{PMP} + \left(p_2^{PMP} - c_2^{PMP} \right) \frac{\partial q_2^{PMP}}{\partial \beta} < 0 \text{, where}$$

$$\frac{\partial c_2^{PMP}}{\partial \beta} = f - e + \frac{\partial p^{PMP}}{\partial \beta};$$

$$\frac{\partial CS^{PMP}}{\partial \beta} = q_1^{PMP} \frac{\partial q_1^{PMP}}{\partial \beta} + q_2^{PMP} \frac{\partial q_2^{PMP}}{\partial \beta} + r \left(q_1^{PMP} \frac{\partial q_2^{PMP}}{\partial \beta} + q_2^{PMP} \frac{\partial q_1^{PMP}}{\partial \beta} \right) > 0;$$

$$\frac{\partial SW^{PMP}}{\partial \beta} = \frac{\partial \pi^{PMP}}{\partial \beta} + \frac{\partial \pi_1^{PMP}}{\partial \beta} + \frac{\partial \pi_2^{PMP}}{\partial \beta} + \frac{\partial CS^{PMP}}{\partial \beta} > 0.$$

(2) The effects of r:

$$\frac{\partial p^{PMP}}{\partial r} = \frac{\{[3(a-e-n)\beta - 8a + 6e + 2f + 8n]r^2 + [-12(a-e-n)\beta + 24a - 16e - 8f - 24n]r + 4(a-e-n)(3\beta-4)\}\beta}{2[r^2 + (4-3\beta)r + 6\beta - 8]^2},$$

if $0 \le r \le r_1$, then $\frac{\partial p^{PMP}}{\partial r} \le 0$; if $r_1 < r \le 1$, then $\frac{\partial p^{PMP}}{\partial r} > 0$.

$$r_1 = \frac{\begin{array}{l} 6(a-e-n)\beta - 4(3a-2e-f-3n) + \\ 2\sqrt{2(3\beta-4)e^2 - 2(3\beta-4)(a+f-n)e + 4f^2 + 2(3\beta-4)(a-n)f + 4(a-n)^2} \end{array}}{3(a-e-n)\beta - 2(4a-3e-f-n)};$$

$$\frac{\partial q_1^{PMP}}{\partial r} = \frac{\begin{array}{l} \{(\beta^2-5\beta-2)(a-e-n)r^3 + [(a-3e+2f-n)\beta^2 + 2(5a+e-6f-5n)\beta - 4(a-e-n)]r^2 + [-2(5a+e- \\ 6f-5n)\beta^2 - 8(a-3e+2f-n)\beta + 32(a-e-n)]r + 8(a+3e-4f-n)\beta^2 + 8(a-7e+6f-n)\beta - 32(a- \\ e-n)\}\{4(4-\beta)r^3 + (9\beta^2-48\beta+48)r^2 - 4(3\beta^2-24\beta+20)r - 12\beta^2 + 40\beta - 32\} + \{[3(a-e-n)r^2 + 2(a- \\ 3e+2f-n)r - 10a - 2e + 12f + 10n]\beta^2 + [-15(a-e-n)r^2 + 4(5a+e-6f-5n)r - 8(a-3e+2f-n)]\beta - \\ 2(a-e-n)(3r^2+4r-16)\}[r^2 + (-3\beta+4)r + 6\beta - 8][(\beta-4)r^2 - 4\beta + 8] \end{array}}{2[r^2 + (-3\beta+4)r + 6\beta - 8]^2[(\beta-4)r^2 - 4\beta + 8]^2} < 0;$$

$$\frac{\partial q_2^{PMP}}{\partial r} = \frac{\begin{array}{l} \{(\beta-1)(a-e-n)r^4 + [-4(a-e-n)\beta^2 + (11a-13e+2f-11n)\beta - 6(a-e-n)]r^3 + 2(\beta-2)[4(a- \\ f-n)\beta - (a-e-n)]r^2 + [2(5a-11e+6f-5n)\beta^2 - 8(5a-7e+2f-5n)\beta + 32(a-e-n)]r - \\ 4(\beta-2)[(5a-3e-2f-5n)\beta - 4(a-e-n)]\}\{4(4-\beta)r^3 + (9\beta^2-48\beta+48)r^2 - 4(3\beta^2-24\beta+20)r - \\ 12\beta^2 + 40\beta - 32\} + 4(\beta-1)(a-e-n)r^3 + 3[-4(a-e-n)\beta^2 + (11a-13e+2f-11n)\beta - 6(a-e-n)]r^2 + \\ 4(\beta-2)[4(a-f-n)\beta - (a-e-n)r + [2(5a-11e+6f-5n)\beta^2 - 8(5a-7e+2f-5n)\beta + \\ 32(a-e-n)]\}[r^2 + (-3\beta+4)r + 6\beta \pm 8][(\beta-4)r^2 - 4\beta + 8] \end{array}}{2[r^2 + (-3\beta+4)r + 6\beta - 8]^2[(\beta-4)r^2 - 4\beta + 8]^2} > 0;$$

$$\frac{\partial p_1^{PMP}}{\partial r}=\frac{\begin{gathered}\{-(\beta-1)(a-e-n)r^5+[4(a-e-n)\beta^2-(9a-13e+2f-11n)\beta-2(a+3e+3n)]r^4+[(-15a+e+8f+9n)\beta^2+\\(55a-7e-16f-23n)\beta-2(17a-e-n)]r^3+[(a+25e-14f+11n)\beta^2-2(21a+29e-14f+15n)\beta+4(13a+\\7e+7n)]r^2+[2(27a-5e-10f-15n)\beta^2-16(8a-e-2f-3n)\beta+64a]r-8(7a+3e-4f-n)\beta^2+8(19a+\\7e-6f+n)\beta-32(3a+e+n)\}\{4(4-\beta)r^3+(9\beta^2-48\beta+48)r^2-4(3\beta^2-24\beta+20)r-12\beta^2+40\beta-32\}+\\\{5(1-\beta)(a-e-n)\beta^2-(9a-13e+2f-11n)\beta-2(a-3e-3n)]r^3+3[(-15a+e+8f+9n)\beta^2+\\(55a-7e-16f-23n)\beta-2(17a-e-n)]r^2+2[(a+25e-14f+11n)\beta^2-2(21a+29e-14f+15n)\beta+26a+\\14e+14n]r+(27a-5e-10f-15n)\beta^2-16(8a-e-2f-3n)\beta+64a]\}[(\beta-4)r^2-4\beta+8][r^2+(-3\beta+4)r+6\beta-8]\end{gathered}}{2[(\beta-4)r^2-4\beta+8]^2[r^2+(-3\beta+4)r+6\beta-8]^2}<0;$$

$$\frac{\partial p_2^{PMP}}{\partial r}=\frac{\begin{gathered}\{[4(a-e-n)\beta\beta-8(3a-2e-2n)\beta+4(5a+3e+3n)]r^3+[3(3a+e+2f+3n)\beta^2-3(11a+11e+10f+21n)\beta+\\6(11a+5e+5n)]r^2+[-4(7a+e-2f-n)\beta^2+4(23a+13e+13n)\beta-8(11a+9e+9n)]r+[(-2(3a-e+10f+\\9n)\beta^2+16(3a+2f+2n)\beta]-64a\}[(\beta-4)r^2-4\beta+8][r^2+(-3\beta+4)r+6\beta-8]-\{[(a-e-n)\beta^2-2(3a-2e-2n)\beta+\\5a+3e+3n]r^4+[(3a+e+2f+3n)\beta^2-(11a+11e+10f+21n)\beta+22a+10e+10n]r^3+[-2(7a+e-2f-n)\beta^2+\\2(23a+13e+13n)\beta-44a-36e-36n]r^2+[(-2(3a-e+10f+9n)\beta^2+16(3a+2f+2n)\beta-64a]r+4(\beta-2)[(7a+\\3e+2f+5n)\beta-4(3a+e+n)]]\}[4(\beta-4)r^3+(-9\beta^2+48\beta-48)r^2+4(3\beta^2-18\beta+20)r+12\beta^2-40\beta+32]\end{gathered}}{2[(\beta-4)r^2-4\beta+8]^2[r^2+(-3\beta+4)r+6\beta-8]^2}<0;$$

$$\frac{\partial \pi^{PMP}}{\partial r}=\left(\frac{\partial p^{PMP}}{\partial r}-\frac{\partial c^{PMP}}{\partial r}\right)\left(q_1^{PMP}+q_2^{PMP}\right)+\left(p^{PMP}-c^{PMP}\right)\left(\frac{\partial q_1^{PMP}}{\partial r}+\frac{\partial q_2^{PMP}}{\partial r}\right)<0,$$

$$\frac{\partial c^{PMP}}{\partial r}=0;$$

$$\frac{\partial \pi_1^{PMP}}{\partial r}=\left(\frac{\partial p_1^{PMP}}{\partial r}-\frac{\partial c_1^{PMP}}{\partial r}\right)q_1^{PMP}+\left(p_1^{PMP}-c_1^{PMP}\right)\frac{\partial q_1^{PMP}}{\partial r}<0\text{, where }\frac{\partial c_1^{PMP}}{\partial r}=\frac{\partial p^{PMP}}{\partial r};$$

$$\frac{\partial \pi_2^{PMP}}{\partial r}=\left(\frac{\partial p_2^{PMP}}{\partial r}-\frac{\partial c_2^{PMP}}{\partial r}\right)q_2^{PMP}+\left(p_2^{PMP}-c_2^{PMP}\right)\frac{\partial q_2^{PMP}}{\partial r}<0\text{, where }\frac{\partial c_2^{PMP}}{\partial r}=\frac{\partial p^{PMP}}{\partial r};$$

$$\frac{\partial CS^{PMP}}{\partial r}=q_1^{PMP}\frac{\partial q_1^{PMP}}{\partial r}+q_2^{PMP}\frac{\partial q_2^{PMP}}{\partial r}+\left(q_1^{PMP}q_2^{PMP}+q_1^{PMP}\frac{\partial q_2^{PMP}}{\partial r}+q_2^{PMP}\frac{\partial q_1^{PMP}}{\partial r}\right)<0;$$

$$\frac{\partial SW^{PMP}}{\partial r}=\frac{\partial \pi^{PMP}}{\partial r}+\frac{\partial \pi_1^{PMP}}{\partial r}+\frac{\partial \pi_2^{PMP}}{\partial r}+\frac{\partial CS^{PMP}}{\partial r}<0.$$

D: Proof of Proposition 4

The effects of r:

$$\frac{\partial p^{PPP}}{\partial r}=0;\ \frac{\partial \pi^{PPP}}{\partial r}=-\frac{(a-e-n)^2}{2(r+2)^2}<0;\ \frac{\partial q_i^{PPP}}{\partial r}=-\frac{a-e-n}{2(r+2)^2}<0;$$

$$\frac{\partial p_i^{PPP}}{\partial r}=-\frac{a-e-n}{2(r+2)^2}<0;\ \frac{\partial \pi_i^{PPP}}{\partial r}=\frac{(a-e-n)^2}{=2(r+2)^3}<0;\ \frac{\partial CS^{PPP}}{\partial r}=-\frac{(a-e-n)^2}{(r+2)^3}<0;$$

$$\frac{\partial SW^{PPP}}{\partial r}=-\frac{(a-e-n)(a-e-n)(r+6)}{(r+2)^3}<0.$$

E: Proof of Corollary 1

$$\pi^{PMP}-\pi^{PPP}=\frac{\begin{array}{l}(r+1)e^2-2(fr-a+2f+n)e+f^2r-a^2+2an+2f^2-n^2)\beta^2+\{-(3r+5)e^2+\\ 2(2fr-a+4f+n)e\left[(a-2f-n)r+3a-4f-3n\right](a-n)\}\beta-(r+2)(2-3e-2n)e\end{array}}{2(\beta-2-r)(\beta-1)(r+2)};$$

If $a_0 < a \le a_1$, then $\pi^{PMP} \le \pi^{PPP}$; if $a > a_1$, then $\pi^{PMP} > \pi^{PPP}$;

$$q_1^{PMP}-q_1^{MPP}=\frac{\begin{array}{l}\{(\beta^2-5\beta-2)(a-e-n)r^3+\left[(a-3e+2f-n)\beta^2+2(5a+e-6f-5n)\beta-4(a-e-n)\right]r^2+\\ 8(a-7e+6f-n)\beta-\left[-2(5a+e-6f-5n)\beta^2-8(a-3e+2f-n)\beta+32(a-e-n)\right]r+\\ 8(a+3e-4f-n)\beta^2+32(a-e-n)\}\left[(3+r)\beta-2(2+r)\right]-2\left[r^2+(-3\beta+4)+6\beta-8\right][(\beta-\\ 4)r^2-4\beta+8]\left[(3+r)r^2-2(2+r)\right]\end{array}}{2\left[r^2+(-3\beta+4)r+6\beta-8\right]\left[(\beta-4)r^2-4\beta+8\right]\left[(3+r)\beta-2(2+r)\right]};$$

If $a_0 < a \le a_2$, then $q_1^{MPP} \le q_1^{PMP}$; if $a > a_2$, then $q_1^{MPP} > q_1^{PMP}$;

$$p_1^{PMP}-p_1^{MPP}=\frac{\begin{array}{l}\{-(\beta-1)(a-e-n)r^5+\left[4(a-e-n)\beta^2-(9a-13e+2f-11n)\beta-2(a+3e+3n)\right]r^4+\\ \left[(-15a+e+8f+9n)\beta^2+(55a-7e-16f-23n)\beta-2(17-e-n)\right]r^3+[(a+25e-14f+11n)\beta^2-\\ 2(21a+29e-14f+15n)\beta+4(13a+7e+7n)\right]r^2+\left[2(27a-5e-10f-15n)\beta^2-16(8a-e-2f-\\ 3n)\beta+64a]r-8(7a+3e-4f-n)\beta^2+8(19a+7e-6f+n)\beta-32(3a+e+n)\}\left[(3+r)\beta-2(2+r)\right]-\\ 2\left[(\beta-4)r^2-4r^2+8\right]\left[r^2+(-3\beta+4)r+6\beta-8\right]\left\{\left[(a-m-n)r+3a-m-n\right]\beta-(a-e-n)r-3a-e-n\right\}\end{array}}{2\left[(\beta-4)r^2-4\beta+8\right]\left[r^2+(-3\beta+4)r+6\beta-8\right]\left[(3+r)\beta-2(2+r)\right]}.$$

If $a_0 < a \le a_3$, then $p_1^{PMP} \le p_1^{MPP}$; if $a > a_2$, then $p_1^{PMP} > p_1^{MPP}$;

$$SW^{PMM}-SW^{PPP}=\frac{\begin{array}{l}4(r+2)^2(e-f)^2\beta^3+\{-3(e-f)^2r^3+(8a-35e+19f-8n)(e-f)r^2+[-101e^2+\\ 2(13a+72f-26n)e-40f^2-32(a-n)f+3(a-n)^2\,]r-85e^2+6(3a+20f-3n)e-\\ 28f^2-32(a-n)f+7(a-n)^2\}\beta^2-2(r+2)\{3(e-f)(a-2e-n)r^2+[-31e^2+(15a+\\ 26f-15n)e+(a-n)(a-13f-n)]r-37e^2+4(4a+7f-4n)e+(3a-14f-3n)(a-\\ n)\}\beta+(r+2)^2(3r+7)e(2a-3e-2n)\end{array}}{-4(r+2)^2(2+r-\beta)^2}.$$

If $a_0 < a \le a_4$, then $SW^{PMM} \le SW^{PPP}$; if $a > a_4$, then $SW^{PMM} > SW^{PPP}$;

$$a_0=\frac{\begin{array}{l}(\beta-1)(e+n)r^5+\left[(-3e-2f-5n)\beta^2+2(4e+3f+7n)\beta-2(e+n)\right]r^4+[-6(e-f)\beta^3+2(19e-11f+\\ 8n)\beta^2+(-63e+8f-55n)\beta+18(e+n)\right]r^3+\left[12(e-f)\beta^3+(-49e+54f+5n)\beta^2+2(25e-18f+\\ 7n)\beta-12(e+n)r^2+\left[24(e-f)\beta^3-2(55e-26f+29n)\beta^2+8(17e-2f+15n)\beta-32(e+n)\right]r-\\ 48(e-f)\beta^3+16(10e-7f+3n)\beta^2-8(19e-6f+13n)\beta+32(e+n)\end{array}}{(\beta-1)r^5+(-5\beta^2+14\beta-2)r^4+(16\beta^2-55\beta+18)r^3+(5\beta^2+14\beta-12)r^2-2(29\beta^2-60\beta+16)r+8(6\beta^2-13\beta+4)};$$

$$a_1=\frac{\begin{array}{l}(\beta-1)^2(e+n)r^4+\left[2(2e-f+n)\beta^2-2(7e-f+n)\beta+4(e+n)\right]r^3+[-(7e+2f+9n)\beta^2+12(e+\\ f+2n)\beta-8(e+n)]r^2-8\beta\left[(2e-f-n)\beta-4(e+n)\right]r+4(3e+2f+5n)\beta^2-16(e+2f+3n)\beta-\\ 2\sqrt{-2(r+2)\beta^2\left[r^2+(-3\beta+4)r+2(3\beta-4)\right](e-f)^2\left[(\beta-4)r^2-4(\beta-2)\right]}\end{array}}{(\beta-1)^2r^4+2(\beta^2-6\beta+2)r^3+(-9\beta^2+24\beta-8)r^2-8(\beta-4)\beta r+4(5\beta-12)\beta};$$

$$a_2=\frac{\begin{array}{l}\left[\left(e+n\right)r^3+2\left(4e-f-3m+6n\right)r^2+\left(27e-22f+5n\right)r+36e-48f+24m-36n\right]\left(r-2\right)\beta^3+\\ \left[\left(-7e+2m-9n\right)r^4+\left(-33e+16f+32m-49n\right)r^3-2\left(17e-42f+36m-61n\right)r^2+16(3e-f-\right.\\ \left.5m+7n)r+216e-272f+160m-216n\right]\beta^2\beta^2+\left[2\left(5e-4m+9n\right)r^4+2\left(23e-12f-16m+27n\right)r^3+\right.\\ \left.4\left(e-20f+20m-39n\right)r^2-32\left(e-f-2m+2n\right)r-160e+192f-128m+160n\right]\beta-4r^2\left(r^2+4r-8\right)\left(e+n\right)\end{array}}{\left(r^4+4r^3-7r^2-22r+24\right)\beta^3-\left(7r^4+17r^3-50r^2-32r+56\right)\beta^2+2\left(5r^4+11r^3-38r^2+16\right)\beta-4r^2\left(r^2+4r-8\right)};$$

$$a_3=\frac{\begin{array}{l}\left[-4\left(e+n\right)r^4+\left(-19e+8f-6m-5n\right)r^3-2\left(5e-13f+3m-11n\right)r^2+\left(45e-10f+24m+11n\right)r+\right.\\ \left.4\left(9e-12f+6m-9n\right)\right]\left(r-2\right)\beta^3+\left[\left(e+n\right)\right]r^6+2\left(12e-f+m+10n\right)r^5+2\left(20e-19f+17m-16n\right)r^4+\\ \left(-127e-24f-40m-111n\right)r^3+\left(-202e+212f-152m+162n\right)r^2+\left(168e+64f+80m+152n\right)r+\\ 216e-272f+160m-216n\Big]\beta^2-\Big[3\left(e+n\right)r^4+\left(37e-4f+8m+25n\right)r^3+\left(26e-40f+40m-18n\right)r^2+\\ \left(-64e-16f-32m-48n\right)r-80e+96f-64m+80n\Big]\left(r^2-2\right)\beta+2r^2\left(r^2+4r-8\right)\left(e+n\right)\left(r^2-2\right)\end{array}}{\begin{array}{c}\left(-4r^5-3r^4+38r^3+3r^2-82r+24\right)\beta^3+\left(r^6+22r^5+2r^4-151r^3+10r^2+232r-56\right)\beta^2+\\ 2\left(-3r^6-33r^5+20r^4+146r^3-44r^2-160r+32\right)\beta+2r^2\left(r^2+4r-8\right)\left(r^2-2\right)\end{array}};$$

$$a_4=\frac{\begin{array}{c}\left[4\left(e-f\right)r^2+\left(13e-16f-3n\right)r+9e-16f-7n\right]\beta^2-\left[3\left(e-f\right)r^2+\left(15e-13f-2n\right)r+16e-14f-2n\right]\left(r+2\right)\beta+\\ 3er^3+19er^2+40er+28e+\sqrt{\left[\left(e-f\right)\beta-e\right]^2\left(3r+7\right)(r+2)^2\left(-\beta+r+2\right)^2\left(3r-4\beta+7\right)}\end{array}}{(\beta-1)^2r^4+2\left(\beta^2-6\beta+2\right)r^3+\left(-9\beta^2+24\beta-8\right)r^2-8\left(\beta-4\right)\beta r+4\left(5\beta-12\right)\beta}.$$

F: Proof of Corollary 2 and Corollary 3

In the Model MPP, by satisfying $\dfrac{\partial SW^{MPP}}{\partial\beta}=0$,

$$\beta^{MPP}=\frac{\begin{array}{l}\left(a-e-6m+5n\right)r+3\left(a-e-4m+3n\right)-\\ \sqrt{\left[\left(a-e-2m+n\right)r+3a-3e-4m+n\right]\left[\left(a-e-6m+5n\right)r+3a-3e-8m+5n\right]}\end{array}}{2\left(3+r\right)\left(m-n\right)}$$

can be obtained, where $F=\sqrt{\left[\left(a-e-2m+n\right)r+3a-3e-4m+n\right]\left[\left(a-e-6m+5n\right)r+3a-3e-8m+5n\right]}$;

$$\frac{\partial\beta^{MPP}}{\partial m}=\frac{\left(a-e-n\right)\left[F-\left(a-e-4m+3n\right)r-3\left(a-e-2m+n\right)\right]}{2F(m-n)^2}<0;$$

$$\frac{\partial\beta^{MPP}}{\partial n}=\frac{\left(a-e-m\right)\left[F-\left(a-e-4m+3n\right)r-3\left(a-e-2m+n\right)\right]}{-2F(m-n)^2}>0;$$

$$\frac{\partial\beta^{MPP}}{\partial e}=\frac{F-\left(a-e-4m+3n\right)r-3\left(a-e-2m+n\right)}{2F\left(m-n\right)}<0;$$

$$\frac{\partial\beta^{MPP}}{\partial r}=\frac{3F-\left(3a-3e-8m+5n\right)r-\left(9a-9e-14m+5n\right)}{F(3+r)^2}<0;$$

in the Model PMM, because $\dfrac{\partial SW^{PMM}}{\partial\beta}>0$, thus when $\beta^{PMM}=\dfrac{4+r-\sqrt{8+8r+r^2}}{4}$, SW^{PMM} is largest, $\dfrac{\partial\beta^{PMM}}{\partial r}=\dfrac{\sqrt{8+8r+r^2}-r-4}{4\sqrt{8+8r+r^2}}<0$; in the Model PMP, because $\dfrac{\partial SW^{PMP}}{\partial\beta}>0$, thus when $\beta^{PMP}=1$, SW^{PMP} can be obtained.