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A FUZZY MULTI-ATTRIBUTE DECISION-MAKING METHOD UNDER RISK WITH UNKNOWN ATTRIBUTE WEIGHTS

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Abstract. This paper aims at solving hybrid multiple attributes decision-making problems under risk with attribute weight known and a new decision approach based on entropy weight and TOPSIS is proposed. First, the risk decision matrix is transformed into the certain decision matrix based on the expectation value. Then, the deviation entropy weight method is used to determine the attribute weights. And according to the definitions of the distance and the positive/negative ideal solutions for different data types, the relative closeness coefficients can be calculated by TOPSIS. Furthermore, the alternatives are ranked by the relative closeness coefficients. Finally, an application case is given to demonstrate the steps and effectiveness of the proposed approach.

Keywords: hybrid decision, risk decision, TOPSIS.

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1. Introduction

The hybrid multiple attribute decision making problems are the multiple attribute decision making problems where the attributes contain both the quantitative index and the qualitative index. The multiple attribute decision making is widely used in the field of society, economy, management, military affairs and engineering technology to solve the problems such as investment decision, project evaluation, economic benefit evaluation and personnel performance appraisal, etc. (Hwang and Yoon 1981; Zavadskas *et al.* 2010a, 2010b; Ginevičius *et al.* 2008; Ginevičius 2009; Liu 2009a, 2009b; Arslan and Aydin 2009). The quantitative indexes of these problems are usually difficult to be quantified accurately, and they often take the fuzzy or incomplete form, so these problems are called the hybrid multiple attribute decision making problems and the attribute values are expressed by different data types, such as precision number, interval number, triangular fuzzy number, linguistic variable. Furthermore, for some decision making problems, the decision-makers often face an uncertain environment and the attribute values of the alternatives are the random variables which change as the natural state, and the decision-makers was uncertain of their real state in the future, but they can give all possible natural states, and they can quantify the randomness by setting the probability distribution. These above decision making problems are called the multiple attribute decision making under risk (Yu *et al.* 2003).

Some decision making problems are both the hybrid multiple attribute decision making problems and the multiple attribute decision making problems under risk, because of the complexity and uncertainty of the decision making problems, so we called these decision making problems the hybrid multiple attribute decision making problems under risk. So the researches on the hybrid multiple attribute decision making problems under risk have not only the important theory significance but also the strong practical value. Yu *et al.* (2003) researched on the hybrid multiple attributes decision making problem where the attribute weights are unknown and the attribute values are the real numbers, and they proposed a correlative mathematics model. Xia and Wu (2004) proposed the TOPSIS method based on hybrid multiple attribute decision making problem under the attribute weight known. Ding *et al.* (2007) researched on the hybrid multiple attribute decision making problems where the attribute values are the hybrid number, such as the real number, the interval number, the linguistic variable and the uncertain linguistic variable, and they proposed a decision making method based on the similarity degree under attribute weight known. Yan *et al.* (2008) also researched on the hybrid multiple attribute decision making problems under the attribute weight unknown, firstly, the maximizing deviation method was used to determine the attribute weight, and then the grey relation method was used to solve the ranking of the alternatives. Wang (2005), Bai *et al.* (2006), Wang and Cui (2007) researched on the hybrid multiple attribute decision making methods from the aspects of the connection number, the possibility degree and the entropy weight, respectively. However, the attribute index under risk wasn't considered in these references (Xia and Wu 2004; Ding *et al.* 2007; Yan *et al.* 2008; Wang 2005; Bai *et al.* 2006; Wang and Cui 2007). Luo and Liu (2004) proposed a grey fuzzy relation method and the double base points method, based on the decision making problem under risk where the attribute weights are unknown and the attribute values are the interval numbers. Yao (2007) proposed an extended TOPSIS method, based on the multiple attribute decision making problems under risk with the continuous random variables. At present, the research on the hybrid multiple attribute decision making problems is less. Rao and Xiao (2006) proposed a dynamic hybrid multiple attribute decision making method under risk based on the grey matrix relation degree, which was aiming at the hybrid multiple attribute decision making problems under risk where the attribute weights were unknown and the attribute values were the real numbers, the interval number and the linguistic fuzzy numbers.

This paper focuses on the hybrid multiple attribute decision making problems under risk with the attribute weight known. Firstly, the risk decision matrix is transformed into the certain decision matrix based on the expectation value; then the deviation entropy weight method is used to determine the attribute weights; finally, the TOPSIS method is used to solve the hybrid decision making problems.

2. The description of the decision making problems

In the hybrid multiple attribute decision making problems under risk, suppose that $A = (a_1, a_2,\ldots, a_n)$ presents the set of the evaluation alternatives, and $C = (c_1, c_2,\ldots, c_n)$ presents the set of the evaluation indexes (or attributes), and $W = (w_1, wc_2,\ldots, w_n)$ represent the attribute weight set, where w_j is the weight of the attribute c_j, and $0 \le w_j \le 1, \sum_{j=1}^{n} w_j = 1$, and the attribute weight values are unknown. For the attribute c_j, there are l_j kinds of possible states $\Theta_j = (\theta_1, \theta_2, \ldots, \theta_{l_j})$, and the probability of the attribute c_j under the state θ_t is p_j^t, where $0 \le p_j^t \le 1, \sum_{t=1}^{l_j} p_j^t = 1$. For the attribute c_j under the natural state θ_t, the attribute value of the alternative a_j is x_{ij}^t, the data type of x_{ij}^t is one of the precision number, the interval number, the triangular fuzzy number, and the linguistic variables (the date is shown in Table 1). The alternatives of the hybrid multiple attribute decision making under risk will be evaluated comprehensively according to these conditions.

Table 1. The decision data of the hybrid multiple attribute decision making under risk

	c_1			c_2					$\ldots$		c_n		
	θ_1	θ_2	$\ldots$	θ_{l_1}	θ_1	θ_2	$\ldots$	θ_{l_2}	$\ldots$	θ_1	θ_2	$\ldots$	θ_{l_n}
	p_1^1	p_1^2	$\ldots$	$p_1^{l_1}$	p_2^1	p_2^2	$\ldots$	$p_2^{l_2}$	$\ldots$	p_n^1	p_n^2	$\ldots$	$p_n^{l_n}$
a_1	x_{11}^1	x_{11}^2	$\ldots$	$x_{11}^{l_1}$	x_{12}^1	x_{12}^2	$\ldots$	$x_{12}^{l_2}$	$\ldots$	x_{1n}^1	x_{1n}^2	$\ldots$	$x_{1n}^{l_n}$
a_2	x_{21}^1	x_{21}^2	$\ldots$	$x_{21}^{l_1}$	x_{22}^1	x_{22}^2	$\ldots$	$x_{22}^{l_2}$	$\ldots$	x_{2n}^1	x_{2n}^2	$\ldots$	$x_{2n}^{l_n}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
a_m	x_{m1}^1	x_{m1}^2	$\ldots$	$x_{m1}^{l_1}$	x_{m2}^1	x_{m2}^2	$\ldots$	$x_{m2}^{l_2}$	$\ldots$	x_{mn}^1	x_{mn}^2	$\ldots$	$x_{mn}^{l_n}$

3. The decision making method and the steps

3.1. Preliminaries

3.1.1. The operational laws of the interval number (Rao and Xiao 2006)

Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$ be two interval numbers, then the operational laws are shown as follows:

$$a + b = [a^L, a^U] + [b^L, b^U] = [a^L + b^L, a^U + b^U], \tag{1}$$

$$a - b = [a^L, a^U] - [b^L, b^U] = [a^L - b^U, a^U - b^L], \tag{2}$$

$$ab \approx [a^L, a^U] \times [b^L, b^U] = [a^L b^L, a^U b^U], \tag{3}$$

$$a / b \approx [a^L, a^U] / [b^L, b^U] = [a^L / b^U, a^U / b^L], \tag{4}$$

$$\lambda a = \lambda [a^L, a^U] = [\lambda a^L, \lambda a^U] \quad \lambda > 0. \tag{5}$$

3.1.2. The operational laws of the triangular fuzzy numbers

Definition 1(Wang and Zhao 2006): let $\tilde{a} = (a^L, a^M, a^U)$ be the triangular fuzzy number, and its membership function $a(x): R \to [0,1]$ is shown as follows:

$$a(x) = \begin{cases} \dfrac{x - a^L}{a^M - a^L}, x \in (a^L, a^M) \\ \dfrac{x - a^U}{a^M - a^U}, x \in (a^M, a^U) \\ 1, \quad x = a^M \\ 0, \quad x \in (-\infty, a^L) \cup (a^M, \infty). \end{cases} \tag{6}$$

The element x of the triangular fuzzy number is real number, and its membership function $a(x)$ represents the degree that element x belongs to the fuzzy set $\tilde{a}$. $a(x)$ is the regular, continued and convex function, and it is composed of the linear non-increasing and non-decreasing part, and it forms a triangle. Generally, $a^L < a^M < a^U$, where a^L and a^U are represent the Lower Bounds element and Upper Bounds element of the fuzzy number, respectively, and the difference value between a^L and a^U represents the fuzzy degree; a^M is the primary element of $\tilde{a}$, and its membership degree is the highest. Specially, if $a^L = a^M = a^U$, then $\tilde{a} = a^M$, thus the triangular fuzzy number degenerates into a real number.

Let $\tilde{a} = [a^L, a^M, a^U]$ and $\tilde{b} = [b^L, b^M, b^U]$ be two triangular fuzzy numbers, then according to the extension principle of the fuzzy sets, the operational laws are shown as follows (Wang and Zhao 2006):

$$\tilde{a} + \tilde{b} = [a^L, a^M, a^U] + [b^L, b^M, b^U] = [a^L + b^L, a^M + b^M, a^U + b^U], \tag{7}$$

$$\tilde{a} - \tilde{b} = (a^L - b^U, a^M - b^M, a^U - b^L), \tag{8}$$

$$\tilde{a}\tilde{b} = [a^L, a^M, a^U][b^L, b^M, b^U] = [a^L b^L, a^M b^M, a^U b^U], \tag{9}$$

$$\lambda \tilde{a} = [\lambda a^L, \lambda a^M, \lambda a^U], \lambda \geq 0, \tag{10}$$

$$\frac{1}{\tilde{a}} = \left(\frac{1}{a^U}, \frac{1}{a^M}, \frac{1}{a^L}\right). \tag{11}$$

3.1.3. The transformation between the linguistic variable and the triangular fuzzy number

The linguistic assessment value is generally choosed from the predefined linguistic assessment set. The linguistic assessment set is an ordered set which is composed of the odd elements, such as the linguistic assessment set S = (very poor, poor, fair, good, very good) which is composed of five elements; the linguistic assessment set S = (very poor, poor, moderately poor, fair, moderately good, good, very good) which is composed of seven elements. When the number of the elements is seven, the corresponding relation between the linguistic variable and the triangular fuzzy number is shown in Table 2.

Table 2. The corresponding relation between the linguistic variable and the triangular fuzzy number (the number of the elements is seven)

Number	Linguistic valuation set S	Triangular fuzzy number
1	very poor	(0,0,0.1)
2	poor	(0,0.1,0.3)
3	moderately poor	(0.1,0.3,0.5)
4	fair	(0.3,0.5,0.7)
5	moderately good	(0.5,0,7,0.9)
6	good	(0.7,0.9,1)
7	very good	(0.9,1,1)

3.2. The decision making method

3.2.1. Transformation of linguistic variables into triangle fuzzy numbers

According to the operational laws of the interval number and the triangular fuzzy number, solve the expectation value of each state in Table 1 in order to transform the risk decision matrix into a certain decision matrix $Z=[z_{ij}]_{m\times n}$, where

$$z_{ij}=\sum_{t=1}^{l_j} p_j^t x_{ij}^t . \tag{12}$$

3.2.2. The normalization of the decision making matrix

Normalize the decision making matrix, in order to eliminate the effect of the different physical dimensions on the decision making result. The most common index (attribute) type are the benefit index (I_1) and the cost index (I_2). The normalized methods are shown as follows:
(1) The normalization method of the real number:

$$r_{ij}=z_{ij}\,/\sqrt{\sum_{i=1}^{m} z_{ij}^2} \quad j\in I_1, \tag{13a}$$

$$r_{ij}=\frac{1}{z_{ij}}\,/\sqrt{\sum_{i=1}^{m}\left(\frac{1}{z_{ij}}\right)^2} \quad j\in I_2. \tag{13b}$$

(2) The normalization method of the interval numbers (Da and Xu 2002):

- suppose that the interval number is expressed by $z_{ij}=[z_{ij}^L, z_{ij}^U]$, after being normalized, $z_{ij}=[z_{ij}^L, z_{ij}^U]$ changes into $r_{ij}=[r_{ij}^L, r_{ij}^U]$, then the normalization method is shown as follows:

$$\begin{cases} r_{ij}^{L} = z_{ij}^{L} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{U})^2} \\ r_{ij}^{U} = z_{ij}^{U} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{L})^2} \end{cases} \quad j \in I_1, \tag{14a}$$

$$\begin{cases} r_{ij}^{L} = (1/z_{ij}^{U}) / \sqrt{\sum_{i=1}^{m} (1/z_{ij}^{L})^2} \\ r_{ij}^{U} = (1/z_{ij}^{L}) / \sqrt{\sum_{i=1}^{m} (1/z_{ij}^{U})^2} \end{cases} \quad j \in I_2. \tag{14b}$$

(3) The normalization method of the triangular fuzzy numbers (Da and Xu 2002):

– suppose that the triangular fuzzy number is expressed by $\left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{r}\right)$, after being normalized, $\left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{r}\right)$ changes into $\left(b_{ij}^{l}, b_{ij}^{m}, b_{ij}^{r}\right)$, then the normalization method is shown as follows:

$$\begin{cases} r_{ij}^{L} = z_{ij}^{L} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{U})^2} \\ r_{ij}^{M} = z_{ij}^{M} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{M})^2} \\ r_{ij}^{U} = z_{ij}^{U} / \sqrt{\sum_{i=1}^{m} (z_{ij}^{L})^2} \end{cases} \quad j \in I_1, \tag{15a}$$

$$\begin{cases} r_{ij}^{L} = (1/z_{ij}^{U}) / \sqrt{\sum_{i=1}^{m} (1/z_{ij}^{L})^2} \\ r_{ij}^{M} = (1/z_{ij}^{M}) / \sqrt{\sum_{i=1}^{m} (1/z_{ij}^{M})^2} \\ r_{ij}^{U} = (1/z_{ij}^{L}) / \sqrt{\sum_{i=1}^{m} (1/z_{ij}^{U})^2} \end{cases} \quad j \in I_2. \tag{15b}$$

3.2.3. The definition of the distance in different data types

(1) Real numbers.

Let a and b be two real numbers, then the distance between a and b is defined as follows:

$$d(a,b) = |a - b|. \tag{16a}$$

(2) Interval numbers.

Let a (a^l,a^r) and b (b^l,b^r) be two interval numbers, then the distance between a and b and is defined as follows:

$$d(a,b)=\frac{\sqrt{2}}{2}\sqrt{(a^L-b^L)^2+(a^U-b^U)^2}. \tag{16b}$$

(3) Triangular fuzzy numbers.

Let a (a^l, a^m, a^r) and b (b^l, b^m, b^r) be two triangular fuzzy numbers, then the distance between a and b and is defined as follows:

$$d(a,b)=\frac{\sqrt{3}}{3}\sqrt{(a^L-b^L)^2+(a^M-b^M)^2+(a^U-b^U)^2}. \tag{16c}$$

3.2.4. The attribute weight

The entropy method firstly appeared in the thermodynamics, and it was introduced into the information theory by Shannon (1948). Nowadays, it has been widely used in engineering, economy, finance, etc. Information entropy is the measurement of the disorder degree of a system (Meng 1989). It can measure the amount of useful information with the data provided. When the difference of the values among the evaluating objects on the same attribute is large, while the entropy is small, it illustrates that this attribute provides more useful information, and the weight of this attribute should be set larger. On the other hand, if the difference is smaller and the entropy is larger, the relative weight would be smaller (Qiu 2002). Hence, the entropy theory is an objective way for the weight determination, and it has been widely used to determine the weight in decision making problems (Hwang and Yoon 1981; Zeleny 1982; Qiu 2002; Liu 2010; Zou *et al.* 2006; Wang and Lee 2009). However, the entropy theory is only used in the classical multiple criteria decision making (MCDM) problems, among which the attribute value is measured in the crisp numbers, and this paper used the entropy method to determine the weight in hybrid types of the attribute value.

For the attribute c_j, we defined that the deviation D_{ij} between the alternative a_j and all other deviation:

$$D_{ij}=\sum_{k=1}^{m}d(r_{ij},r_{kj}) \quad (i=1,2,\cdots,m; j=1,2,\cdots,n). \tag{17}$$

For the attribute c_j, we defined that the total deviation D_{ij} between each alternative and all other alternative:

$$D_j=\sum_{i=1}^{m}D_{ij}=\sum_{i=1}^{m}\sum_{k=1}^{m}d(r_{ij},r_{kj}) \quad (j=1,2,\cdots,n). \tag{18}$$

For the attribute c_j, the decision making information can be expressed by the following entropy E_j, :

$$E_j=-K\sum_{i=1}^{m}\frac{D_{ij}}{D_j}\ln\frac{D_{ij}}{D_j} \quad (1 \le j \le n), \tag{19}$$

where $K=1/\ln m$, and m is the number of the alternatives. Suppose that $\frac{D_{ij}}{D_j}=0$, and $\frac{D_{ij}}{D_j}\ln\frac{D_{ij}}{D_j}=0$.

The difference degree of attribute c_j can be calculated as follows:

$$G_j = 1 - E_j\,(1 \le j \le n)\,. \tag{20}$$

The entropy weight w_j can be calculated as follows:

$$w_j = G_j \,/ \sum_{j=1}^{n} G_j (1 \le j \le n)\,. \tag{21}$$

3.2.5. The weighting hybrid matrix

According to the entropy weight, calculate the weighting normalized matrix V:

$$V = (v_{ij})_{m \times n} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix}. \tag{22}$$

3.2.6. Use TOPSIS to evaluate the alternatives

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) is a practical and useful technique for ranking and selection of a number of possible alternatives through measuring Euclidean distances. TOPSIS was first developed by Hwang and Yoon (1981). It bases on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS is widely used in multiple attribute decision making, and it has been extended with respect to various attributes by using the fuzzy numbers instead of the precise numbers. Jahanshahloo *et al.* (2006a, 2006b) extended TOPSIS to solve the decision making problems where the attribute value take the from of the interval number and the fuzzy number. Chen and Tsao (2008) extended the TOPSIS method based on the interval-valued fuzzy sets in decision analysis. This paper is to extend the TOPSIS method to solve the decision making problems with the hybrid types of the attribute value.

(1) The positive / negative ideal solution of the alternative:

– suppose that G^+ and G^- represent the positive and negative ideal solution, respectively. For the attribute c_j, g_j^+ and g_j^- represent the attribute value of the positive and negative ideal solution, respectively. Then the positive and negative ideal solution for different data types is shown as follows:

(i) Real number type.

If the attribute value of the attribute c_j is the real number v_{ij}, then:

$$g_j^+ = \max_i(v_{ij}), \quad g_j^- = \min_i(v_{ij}). \tag{23a}$$

(ii) Interval number type.

If the attribute value of the attribute c_j is interval number $[v_{ij}^L, v_{ij}^U]$, then:

$$\begin{aligned} g_j^+ &= [g_j^{+L}, g_j^{+U}] = [\max_i(v_{ij}^L), \max_i(v_{ij}^U)] \\ g_j^- &= [g_j^{-L}, g_j^{-U}] = [\min_i(v_{ij}^L), \min_i(v_{ij}^U)]\,. \end{aligned} \tag{23b}$$

(iii) Triangular fuzzy number type.

If the attribute value of attribute c_j is the triangular fuzzy number $[v_{ij}^L, v_{ij}^M, v_{ij}^U]$, then

$$g_j^+ = [g_j^{+L}, g_j^{+M}, g_j^{+U}] = [\max_i(v_{ij}^L), \max_i(v_{ij}^M), \max_i(v_{ij}^U)]$$

$$g_j^- = [g_j^{-L}, g_j^{-M}, g_j^{-U}] = [\min_i(v_{ij}^L), \min_i(v_{ij}^M), \min_i(v_{ij}^U)]. \quad (23c)$$

(2) Calculate the distance between each alternative and the positive / negative solution, respectively:

$$L_i^+ = L(a_i, G^+) = \sqrt{(d(v_{i1}, g_1^+))^2 + (d(v_{i2}, g_2^+))^2 + \cdots + (d(v_{in}, g_n^+))^2}, \quad (24a)$$

$$L_i^- = L(a_i, G^-) = \sqrt{(d(v_{i1}, g_1^-))^2 + (d(v_{i2}, g_2^-))^2 + \cdots + (d(v_{in}, g_n^-))^2}, \quad (24b)$$

where $d(v_{i1}, g_1^+), d(v_{i2}, g_2^+), \cdots, d(v_{in}, g_n^+)$ and $d(v_{i1}, g_1^-), d(v_{i2}, g_2^-), \cdots, d(v_{in}, g_n^-)$ use the different data type of $(v_{i1}, g_1^+), (v_{i2}, g_2^+), \cdots, (v_{in}, g_n^+)$ and $(v_{i1}, g_1^-), (v_{i2}, g_2^-), \cdots, (v_{in}, g_n^-)$, respectively, and they are calculated with formula (16a),(16b),(16c).

(3) Determine the relative closeness degree.

The relative closeness degree between each alternative and ideal solution is shown as follows:

$$C_i = \frac{L_i^-}{L_i^+ + L_i^-} \quad (i = 1, 2, \cdots, m). \quad (25)$$

(4) Rank the order of the alternatives.

The evaluation alternatives can be ranked according to the value of the relative closeness degree, and the bigger the relative closeness degree is, the better the alternative is.

4. Application case

An enterprise plans to set a new factory. Suppose that the enterprise will choose an optimized alternative from three alternatives a_1, a_2, and a_3. Suppose that there are four attributes c_1, c_2, c_3 and c_4: the direct benefits c_1, the indirect benefits c_2, the social benefits c_3 and the pollution loss c_4. Market forecasts that direct benefits $c_1$and indirect benefits c_2 have four natural states: very good (θ_1), good (θ_2), fair(θ_3) and poor (θ_2); social benefits c_3 and pollution loss c_4 have three natural states: very good (θ_1), good (θ_2), fair(θ_3). Where the direct benefits c_1 is expressed by the real number; indirect benefits c_2 is expressed by the interval number; social benefits c_3 is expressed by the linguistic variable shown in Table 2; pollution loss c_4 is expressed by the triangular fuzzy number. The decision data of each attribute is shown in Table 3.

Table 3. The decision data of each attribute

	c_1				c_2			
	θ_1	θ_2	θ_3	θ_4	θ_1	θ_2	θ_3	θ_4
	0.1	0.3	0.4	0.2	0.1	0.2	0.4	0.3
a_1	25	23	28	30	[95,105]	[95,105]	[95,105]	[95,105]
a_2	25	22	26	22	[90,116]	[97,113]	[97,113]	[97,113]
a_3	28	30	20	18	[90,112]	[97,109]	[104,116]	[104,116]

Table 4. The decision data of each attribute (cont.)

	c_3			c_4		
	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3
	0.3	0.3	0.4	0.3	0.2	0.5
a_1	good	good	fair	[190,200,210]	[195,205,215]	[200,210,220]
a_2	very good	fair	poor	[210,220,230]	[215,225,235]	[185,195,205]
a_3	M-good*	good	M-poor**	[175,195,205]	[235,245,255]	[195,215,235]

* M-good = moderately good ,** M-poor = moderately poor.

The decision steps are shown as follows:

(1) Transform the risk matrix into the certain matrix

$$Z = \begin{bmatrix} 26.6,[114.0,126.0] & [0.54,0.74,0.88] & [196.0,206.0,216.0] \\ 23.9,[115.7,135.9] & [0.36,0.49,0.63] & [198.5,208.5,218.5] \\ 23.4,[120.6,136.0] & [0.40,0.60,0.77] & [197.0,215.0,230.0] \end{bmatrix};$$

(2) Normalize the decision matrix

$$R = \begin{bmatrix} 0.6224, [0.4959, 0.6228] & [0.4066,0.6907,1.1543] & [0.5107,0.5667,0.6325] \\ 0.5593, [0.5033, 0.6718] & [0.2710,0.4574,0.8264] & [0.5172,0.5736,0.6398] \\ 0.5476, [0.5246, 0.6723] & [0.3012,0.5601,1.0100] & [0.5133,0.5915,0.6735] \end{bmatrix};$$

(3) Calculate the entropy weight

$$w = (0.3870, 0.1692, 0.1639, 0.2800);$$

(4) Calculate the weighting hybrid matrix

$$V = \begin{bmatrix} 0.2409\ , [0.0839,0.1054]\ , [0.0666,0.1132,0.1891]\ , [0.1430,0.1587,0.1771] \\ 0.2164\ , [0.0852,0.1137]\ , [0.0444,0.0749,0.1354]\ , [0.1448,0.1606,0.1791] \\ 0.2119\ , [0.0888,0.1137]\ , [0.0493,0.0918,0.1655]\ , [0.1437,0.1656,0.1886] \end{bmatrix};$$

(5) Solve the positive and negative ideal solution

$$G^+ = (0.2409\ ,[0.0888,0.1137]\ ,[0.0666,0.1132,0.1891]\ ,[0.1448,0.1656,0.1886]);$$

$$G^- = (0.2119\ ,[0.0839,0.1054], [0.0444,0.0749,0.1354]\ ,[0.1430,0.1587,0.1771]);$$

(6) Calculate the distances between each alternative and the positive/negative ideal solution

$$L^+ = (0.0147, 0.0733, 0.0506);$$

$$L^- = (0.0692, 0.0124, 0.0347);$$

(7) Calculate the relative closeness degree

$$C = (0.8252, 0.1445, 0.4070);$$

(8) Rank the order of the alternatives

According to the values of relative closeness degree, the ranking of the alternatives is: $a_1 \succ a_3 \succ a_2$.

(9) Verify the effectiveness of this method

In order to verify the effectiveness of this method, we use the gray correlation method proposed by Rao and Xiao (2006) to re-rank the alternatives, and the ranking result is: $a_1 \succ a_3 \succ a_2$. It is the same as the result ranked by the method proposed in this paper, so we think the method proposed in this paper is effective. In addition, compared with method proposed by Rao and Xiao (2006), the TOPSIS method proposed in this paper is simpler in computing, and this is also the reason that TOPSIS method is more commonly used in the decision making problems.

(10) The sensitivity analysis

When we add or reduce the number of alternatives, original ranking results may be changed in TOPSIS method, that is, the reverse order problem is produced (Li 2008). In order to analysis the sensitivity of this sample which is to identify the optimal alternative, we remove the worst alternative a_2, and re-rank for a_1 and a_3 by method proposed in this paper, then check whether there is the reverse order problem.

After removing the worst alternative a_2, we get the entropy weight

$$w = (0.25, 0.25, 0.25, 0.25).$$

Then we get the relative closeness degree after a series of calculating steps

$$C = (0.7379, 0.2621).$$

So, the ranking of the alternatives is: $a_1 \succ a_3$.

Obviously, before and after removing the worst alternative a_2, they are the same as ranking result, that is, the reverse order problem does not exist in this sample.

5. Conclusions

The hybrid multiple attribute decision making problems under risk are more consistent with the realistic situation, and they are widely applied. In this paper, the hybrid multiple attribute decision making method under risk based on entropy weight and TOPSIS is presented, and the decision making steps are given. The definition of the method is definite and it is easy to understand. The method can solve the risk decision making problems based on many data types, including the precision number, the interval number, the fuzzy number and the linguistic variable. Compared with the method proposed by Rao and Xiao (2006), the method proposed in this paper is simpler in computing, and it enriches and develops the theory and method of the hybrid decision making under risk. But in this paper, the expectation value method is adopted to transform the risk decision making problems into the certain ones when solving risk decision making problem, and it is a very simple method. So other transform methods will be researched continuously in the future.

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References

Arslan, G.; Aydin, O. 2009. A new software development for fuzzy multicriteria decision making, *Technological and Economic Development of Economy* 15(2): 197–212. doi:10.3846/1392-8619.2009.15.197-212

Bai, M. G.; Zhu, J. F.; Yao, Y. 2006. Multiple Mixed –attribute Decision Making Method Based on Possibility Degree, *Commercial Research* (14): 19–21.

Chen, T. Y.; Tsao, C. Y. 2008. The interval-valued fuzzy TOPSIS method and experimental analysis, *Fuzzy Sets and Systems* 15: 1410–1428. doi:10.1016/j.fss.2007.11.004

Da, Q. L.; Xu, Z. S. 2002. Single-objective optimization model in uncertain multi-attribute decision making, *Journal of Systems Engineering* 17(2): 50–55.

Ding, C. M.; Li, F.; Qi, H. 2007.Technique of hybrid multiple attribute decision making based on similarity degree to ideal solution, *Systems Engineering and Electronics* 29(5): 737–740.

Ginevičius, R.; Podvezko, V.; Raslanas, S. 2008. Evaluating the alternative solutions of wall insulation by multi-criteria methods, *Journal of Civil Engineering and Management* 14(4): 217–226. doi:10.3846/1392-3730.2008.14.20

Ginevičius, R. 2009. Quantitative evaluation of unrelated diversification of enterprise activities, *Journal of Civil Engineering and Management* 15(1): 105–111. doi:10.3846/1392-3730.2009.15.105-111

Hwang, C. L.; Yoon, K. 1981. *Multiple Attribute Decision Making: Methods and Application*. Springer, New York.

Jahanshahloo, G. R.; Hosseinzadeh Lotfi, F.; Izadikhah, M. 2006a. An algorithmic method to extend TOPSIS for decision making problems with interval data, *Applied Mathematics and Computation* 175: 1375–1384. doi:10.1016/j.amc.2005.08.048

Jahanshahloo, G. R.; Hosseinzadeh Lotfi, F.; Izadikhah, M. 2006b. Extension of the TOPSIS method for decision making problems with fuzzy data, *Appl. Math. Comput.* 181: 1544–1551. doi:10.1016/j.amc.2006.02.057

Liu, P. D. 2009a. Multi-Attribute Decision making Method Research Based on Interval Vague Set and TOPSIS Method, *Technological and Economic Development of Economy* 15(3): 453–463. doi:10.3846/1392-8619.2009.15.453-463

Liu, P. D. 2009b. A novel method for hybrid multiple attribute decision making, *Knowledge-Based Systems* 22(5): 388–391.

Liu, P. D. 2010.Research on the Supplier Selection of Supply Chain Based on Entropy Weight and Improved ELECTRE-III Method, *International Journal of Production Research.* First published on: 17 February 2010 (iFirst).

Li, W. 2008. *Evaluation and Sensitivity analysis Based on Multiple Attribute Decision-Making*. The master thesis of Donghua university, 33–63.

Luo, D.; Liu, S. F. 2004. Research on grey multi-criteria risk decision making method, *Systems Engineering and Electronics* 26(8): 1057–1060.

Meng, Q. S .1989. *Information theory.* Xi'An Jiaotong : Xi'An Jiaotong University Press.

Qiu, W. H. 2002. *Management decision and applied entropy*. Beijing: China Machine Press.

Rao, C. J.; Xiao, X. P. 2006. Method of grey matrix relative degree for dynamic hybrid multi-attribute decision making under risk, *Systems Engineering and Electronics* 28(9): 1353–1357

Shannon, C. E. 1948. A mathematical theory of communications, *Bell Systems Technical Journal* 27(3): 379–423.

Wang, T. C.; Lee, H. D. 2009. Developing a fuzzy TOPSIS approach based on subjective weights and objective weights, *Expert Systems with Applications* 36: 8980–8985. doi:10.1016/j.eswa.2008.11.035

Wang, W.; Cui, M. M. 2007. A Technique of Entropy for Hybrid Multiple Attribute Decision Making Problems, *Mathematics in Practice and Theory* 37(3): 64–68.

Wang, X. 2005. Study for Multi-attribute Mixed Decision Making Model Basing on the Connection Number, *Journal of Tianjin University of Science and Technology* 20(3): 50–53.

Wang, X. Z.; Zhao, Y. Q. 2006. A New Operator for Triangularr Fuzzy Numbers with Application in Weighted Fuzzy Reasoning, *Fuzzy Systems and Mathematics* 20(1): 67–71.

Xia,Y. Q.; Wu, Q. Z. 2004. A technique of order preference by similarity to ideal solution for hybrid multiple attribute decision making problems, *Journal Of Systems Engineering* 19(6): 630–634.

Yan, S. L.; Yang, W. C.; Xiao, X. P. 2008. The Hybrid Multiple Attributes Decision making Method with Attribute Weight Unknown, *Statistics and Decision* (1): 16–18.

Yao, S. B. 2007. TOPSIS of Continuous Multiple Attribute Decision making Problems under Risk, *Statistics and Decision* (7): 10–11.

Yu, Y. B.; Wang, B. D.; Liu, P. 2003. Risky Multi-objective Decision making Theory and Its Application, *Chinese Journal of Management Science* 11(6): 9–13.

Zavadskas, E. K.; Kaklauskas, A.; Turskis, Z.; Tamošaitienė, J. 2008a. Selection of the Effective Dwelling House Walls by Applying Attributes Values Determined at Intervals, *Journal of Civil Engineering and Management* 14(2): 85–93. doi:10.3846/1392-3730.2008.14.3

Zavadskas, E. K.; Turskis, Z.; Tamošaitiene, J. 2010a. Risk assessment of construction projects, *Journal of Civil Engineering and Management* 16(1): 33–46. doi:10.3846/jcem.2010.03

Zavadskas, E. K.; Vilutiene, T.; Turskis, Z.; Tamosaitiene, J. 2010b. Contractor selection for construction works by applying SAW-G and TOPSIS grey techniques, *Journal of Business Economics and Management* 11(1): 34-–55. doi:10.3846/jbem.2010.03

Zeleny, M. 1982. *Multiple Criteria Decision Making*. New York: McGraw Hill.

Zou, Z. H.; Yun, Y.; Sun, J. N. 2006. Entropy method for determination of weight of evaluating in fuzzy synthetic evaluation for water quality assessment indicators, *Journal of Environmental Sciences* 18(5): 1020–1023. doi:10.1016/S1001-0742(06)60032-6

NERAIŠKUSIS MAŽESNĖS RIZIKOS DAUGIATIKSLIS SPRENDIMŲ PRIĖMIMO METODAS SU NEŽINOMAIS PRISKIRIAMAIS REIKŠMINGUMAIS

Z. Han, P. Liu

Santrauka. Šiame straipsnyje siekiama išspręsti mišrias mažesnės rizikos daugiatiksles sprendimo priėmimo problemas su žinomu priskiriamu reikšmingumu bei yra siūlomas naujas sprendimų priėmimo metodas grindžiamas entropijos reikšmingumu ir TOPSIS. Pirmiausia, rizikos sprendimų matrica yra transformuojama į tam tikrą sprendimų matricą, grindžiamą galimybės verte. Tuomet yra naudojamas entropijos reikšmingumo nuokrypio metodas norint nustatyti priskiriamą reikšmingumą. Atsižvelgiant į atstumo apibrėžimus ir teigiamus / neigiamus idealius sprendimus skirtingiems duomenų tipams, santykinio artumo koeficientas gali būti apskaičiuojami remiantis TOPSIS. Be to, alternatyvos yra reitinguojamos pagal santykinio artumo koeficientus. Galiausiai, yra pateiktas pritaikymo atvejis, siekiant parodyti visus žingsnius ir siūlomo metodo veiksmingumą.

Reikšminiai žodžiai: mišrus sprendimas, rizikos sprendimas, TOPSIS.

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