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THE ANALYSIS OF THE QUALITY OF THE RESULTS OBTAINED WITH THE METHODS OF MULTI-CRITERIA DECISIONS

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Abstract. For many decades we have been dealing with problems of multi-criteria decisions. Numerous methods have been developed in this field and new methods are continuously being created. In the light of a great number of methods currently proposed, it is difficult to gain a profound overview. Comparisons on the performance of various methods are done to a small extent only. When applying the methods, in some cases many mathematical operations are performed, which renders it impossible to sufficiently assess their effect with such a complexity. It is therefore aimed at analysing the peculiarities of several methods in a critical review and to give hints for possible consequences. The analysis is primarily concentrated on the normalisation of indices in the mapping to the interval [1; 0] or [1; ~ 0]. This includes linear functions (the relative difference and the calculation with interval boundaries) and non-linear functions (the hyperbolic function, the quadratic and cubic function, the square root and the logarithmic function). With the critical review, the possibility is offered to the decision maker to better assess the quality of his solution.

Keywords: multiple-criteria evaluation, mapping functions, normalisation, optimisation.

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1. General remarks

For many decades we have been dealing with the problems of multi-criteria decisions (Peldschus *et al.* 1983; Fiedler *et al.* 1986). Numerous methods have been developed in this field (Hwang and Yoon 1981; Figueira *et al.* 2005; Zavadskas *et al.* 2006; Zavadskas, Vaidogas 2008; Morkvėnas *et al.* 2008; Jakimavičius, Burinskienė 2007; Maskeliūnaitė *et al.* 2009) and new methods (Ustinovichius *et al.* 2007; Brauers and Zavadskas 2006; Brauers *et al.* 2008; Peldschus and Zavadskas 2005; Zavadskas *et al.* 2007, 2008a, b; Ginevicius and Podvezko

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2008) are continuously being created. In the light of the great number of methods currently proposed, it is difficult to gain a profound overview. Comparisons on the performance of the various methods are done to a small extent only.

When applying the methods, in some cases many mathematical operations are performed, which renders it impossible to sufficiently assess their effect with such a complexity. Of course, a numerical result is obtained every time, but an assessment of the quality of the results is not feasible. Civil engineers know that in static calculations a check of the equilibrium between external forces (loads) and internal forces (determined according to the developed theory) is performed. If then the sum of all vertical forces, the sum of all horizontal forces and the sum of all moments are equal to zero, an equilibrium between external and internal forces is assumed and the result is accepted as the correct solution.

This option does not exist for the methods of multi-criteria decisions. Therefore, it is of particular importance that we critically review the applied methods. Generally speaking, it can be assumed that with the selection of a method the result may be influenced (Peldschus 2007). For this reason, the conditions for the application of the various methods should be analysed in detail. Only after ascertaining that the conditions for the application of a certain method are fulfilled, the method can be used and the results can be trusted. A numerical verification of the results is according to today's knowledge not feasible.

In a detailed analysis of the problems, a distinction has to be made between the calculation of the characteristic values and the solution methods.

2. Calculation of the characteristic values

Characteristic values are required for the application of the methods of multi-criteria decisions. In the simplest case, dimension-less assessment values according to a point system are used. These are defined on a scale and incorporate a great subjective influence. A better option is ratio values which are based on real data. These ratio values refer to the respective optimal value and are an expression of the effectiveness of the specific characteristic value. The ratio values are mapped to the dimension-less interval [1; 0] or [1; ~ 0] (Zavadskas *et al.* 2008a). By doing so, the difficulties emerging from the different dimensions of the characteristic values are avoided. At the same time the discrepancies stemming from the different magnitudes of the characteristic values are eliminated. For the mapping to the dimension-less interval (normalisation) (Zavadskas *et al.* 2003; Ginevičius 2008; Bhangale *et al.* 2004; Wang and Elhag 2006; Shin *et al.* 2007; Milani *et al.* 2005; Turskis *et al.* 2009; Zavadskas and Turskis 2008) several functions are used.

2.1. Linear functions

2.1.1. The relative deviation

Jüttler (1966) generated dimension-less values for the solution of multiple criteria decisions based on the idea of the relative deviation. Thus, the author was able to consider maximisation as well as minimisation problems at the same time. This idea of the relative deviation was primarily developed for linear optimisation problems. For that purpose, for the *k* linear optimisation problems

$$Z_j = \underline{c}'_j \, \underline{x} \rightarrow \max, j = 1, 2, ..., k$$

$$\underline{A}\underline{x} = \underline{b}$$

$$x \geq 0$$

the decision table was calculated. The optimal solutions are named $\underline{x}_j^*$, j = 1, 2, ..., k. The coefficients for the decision matrix are determined as the relative deviation in terms of the respective optimal value.

$$g_{ij} = \frac{\left| Z_j(\underline{x}_j^*) - Z_j(\underline{x}_i^*) \right|}{Z_j(\underline{x}_j^*)}, \quad i, j = 1, 2, ..., k. \tag{1}$$

The term g_{ij} determined this way, also allows the admission of minimisation problems among the *k* linear optimisation problems, if the condition $Z_j(\underline{x}_j^*) > 0$ is fulfilled for all j = 1, 2, ..., *k*, which is usually the case in practical applications.

The values g_{ij} constitute a quadratic matrix $\underline{G}$ of the order k and they are thus available for the solution of the multiple criteria decision problem. For the target function of the form $Z \rightarrow \max$ the values g_{ij} = 0, ..., 1 are obtained. For the target function of the form $Z \rightarrow \min$ the values $g_{ij} = 0, \ldots, \infty$ are obtained.

From that it can be concluded that the effectiveness of functions of the form $Z \rightarrow \min$ may be much higher than that for functions of the form $Z \rightarrow \max$.

The advantage is that the values g_{ij} are an expression of the "quality" of the several solutions with respect to the target function. They are dimension-less and hence comparable to each other.

The disadvantage is that the problem always needs to be the one, for which the k linear optimisation problems exist under equal side conditions.

2.1.2. The method of Koerth

This method is a continuing development from the idea of the relative deviation presented by Juettler. Körth (1969) enhanced the method in the sense that apart from the k optimal solutions he also includes all further base solutions.

From the relative deviations new elements are calculated, which represent the ratio in terms of the optimal value k of the function.

$$a_{ij} = 1 - \frac{\left| Z_j(\underline{x}_j^*) - Z_j(\underline{x}_i^*) \right|}{Z_j(\underline{x}_j^*)}, \quad i, j = 1, 2, ..., k. \tag{2}$$

Also in this case the condition $Z_j(\underline{x}_j^*) > 0$ must be fulfilled for all j = 1, 2, ..., *k*.

The advantage here is that the values a_{ij} are dimension-less and therefore comparable to each other.

On the downside the problem must again be the one for which k linear optimisation problems exist under equal side conditions. For target functions of the form $Z \to \max$ the values are contained in the interval [1, 0]. For target functions of the form $Z \to \min$ the values a_{ij} are only contained in the interval [1, 0], if $Z_j(\underline{x}_j^*) \leq 2Z_j(\underline{x}_j^*)$ for $i = 1, 2, ..., k$, because otherwise negative values occur in the matrix and the condition $a_{ij} \geq 0$ is no longer fulfilled. Hence, the application possibilities for target functions of the form $Z \to \min$ are reduced.

2.1.3. Calculation with interval boundaries

Interval boundaries are implied by several authors (Weitendorf 1976; Brauers and Zavadskas 2006). As a general formulation the following can be given:

$$q_i = \frac{Q_i - Q_{iu}}{Q_{io} - Q_{iu}}, \text{ if } Q_i \text{ shall be maximised,}$$

$$q_i = \frac{Q_{io} - Q_i}{Q_{io} - Q_{iu}}, \text{ if } Q_i \text{ shall be minimised,} \tag{3}$$

Q_{iu} – largest value, Q_i – smallest value.

The elements q_i are all positive and they are mapped to the interval [0; 1]. The initial values are confined by Q_{io} and Q_{iu}.

The advantage in this case is that n different variants with m target functions can be considered.

A disadvantage is the limitation of the values Q_{io} and Q_{iu} by the selection of the variants so that the solution can be influenced in repeated calculations under consideration of further, also less favourable variants.

2.2. Non-linear functions

2.2.1. Hyperbolic functions

Stopp (1975) uses a linear function for the minimisation and a hyperbolic function for the maximisation.

$$a_{ik} = \begin{cases} \dfrac{100 u_{ik}}{\max\limits_i u_{ik}} & \text{, if } \max\limits_i u_{ik} \text{ is favourable,} \\ \dfrac{100 \min\limits_i u_{ik}}{u_{ik}} & \text{, if } \min\limits_i u_{ik} \text{ is favourable.} \end{cases} \tag{4}$$

The values a_{ik} are mapped to the interval [1; 0] for the maximisation and to the interval [1; ~ 0] for the minimisation.

The advantage of this method is that n different variants and m criteria can be considered. The values a_{ik} are an expression of the ratio to the optimal value in both, the minimisation and the maximisation. This ratio is not altered by adding or removing variants.

On the downside, the values a_{ik} are more reduced in the minimisation than in the maximisation with the same relative change of the optimal value.

These differences stem from the application of different functions. For the maximisation the function $(100/a)\,x_i$ has the characteristics of a linear function. For the minimisation the function $(100a)\,/\,x_i$ has the characteristics of a hyperbole. Hence, an unintentional weighting between maximisation and minimisation is created.

2.2.2. Quadratic and cubic functions

For the solution of optimisation problems in production processes, dimension-less values are required to fulfill the following demands:

- The values must express the ratio to the optimal value.
- The ratio value shall not be dependent on the type of the matrix.
- For the same relative variation, the values must be approximately equal for minimisation and maximisation.
- For a minimisation objective, a useful value must be obtainable even for a multiple of the minimal value.
- The optimal values may occur at any position in the matrix.

In order to fulfil the above-mentioned requirements, the following formula was presented (Peldschus 1986, 2008):

$$b_{ij} = \left(\frac{\min_i a_{ij}}{a_{ij}} \right)^3, \text{ if } \min_i a_{ij} \text{ is favourable,}$$

$$b_{ij} = \left(\frac{a_{ij}}{\max_i a_{ij}} \right)^2, \text{ if } \max_i a_{ij} \text{ is favourable.} \qquad (5)$$

Here a quadratic function is used for the maximisation and a cubic function for the minimisation.

The advantage is that no limitation of the initial values is introduced. Even for the multiple of the minimal value, which is the case in the minimisation, an adaptation of the calculated values is obtained. By the application of non-linear functions, the effectiveness of the optimal values is emphasised.

The disadvantage here is that the calculated values are more attenuated and therefore non-optimal values become less important.

2.2.3. Square root

From the theory of vector analysis the following formula (Zavadskas *et al.* 2006; Brauers *et al.* 2008; Ginevičius *et al.* 2008) was adopted:

$$b_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}, \text{ if } \max_i a_{ij} \text{ is favourable,}$$

$$b_{ij} = 1 - \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}, \text{ if } \min_i a_{ij} \text{ is favourable.} \quad (6)$$

It is advantageous that n different variants and m criteria can be considered. The values a_{ik} are an expression of the ratio to the optimal value in the maximization as well as in the minimisation. No limitation of the initial values is introduced. Also for a multiple of the minimal value a mapping to the interval [1; 0] is obtained.

On the downside, a deformation of the problem is made, whereby the interval [1; 0] is not filled evenly. The deformation is dependent on the optimisation target. For the maximisation, a higher concentration towards the value zero is obtained, whereas a higher concentration towards the value 1 is obtained for the minimisation. If further variants are included in the analysis, the transformed characteristic values are changed and they can thus influence the solution.

2.2.4. Logarithmic function

In a recent development, the logarithmic function was used (Zavadskas and Turskis 2008; Turskis *et al.* 2009):

$$b_{ij} = \frac{\ln(a_{ij})}{\ln(\prod_{i=1}^{n} a_{ij})}, \text{ if } \max_i a_{ij} \text{ is favourable,}$$

$$b_{ij} = \frac{1 - \frac{\ln a_{ij}}{\ln(\prod_{i=1}^{n} a_{ij})}}{n-1}, \text{ if } \min_i a_{ij} \text{ is favourable.} \quad (7)$$

For the calculation of the characteristic values the problem is deformed. The deformations exhibit a certain analogy to the results obtained using the square root.

The advantage here is that n variants and m criteria can be considered. The values a_{ik} are an expression of the ratio to the optimal value in both the minimisation and the maximisation. No limitation of the initial values is introduced. Also for multiples of the minimal value, a mapping to the interval [1; 0] is obtained.

The disadvantage is that the interval [1; 0] is not filled evenly. For the maximisation, a higher concentration towards the value zero is obtained, whereas a higher concentration towards the value 1 is obtained for the minimisation. The values resulting for the minimisation by division by $(n - 1)$ are remarkably smaller. Thus, an essential difference in the significance is created between maximisation and minimisation. If further variants are included in the analysis, the transformed characteristic values are changed and they can thus influence the solution.

2.4. Analysis

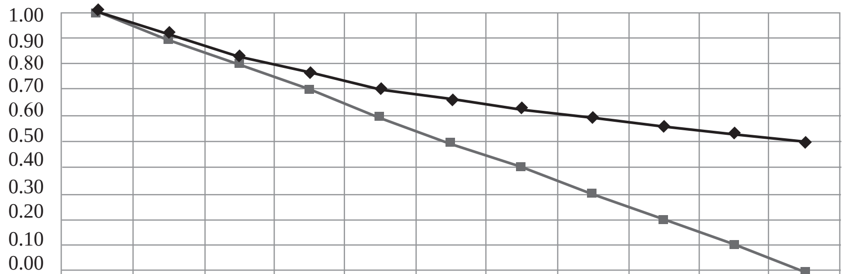
In a numerical analysis the differences shall be investigated. For that purpose, the optimal values are altered in steps of 10% and the transformed values are put in Table 1.

Table 1. Transformed values

Result acc. to formula		Optimal value	Alteration of the optimal value by											
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	150%	200%
Juettler	max	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	–	–
	min	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1,5	2
Koerth	max	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0	–	–
	min	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0	–0.5	–1
Interval boundaries	max	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	–	–
	min	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.75	1
Stopp	max	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0	–	–
	min	1	0.91	0.83	0.77	0.71	0.67	0.63	0.59	0.56	0.53	0,5	0,4	0.33
Peldschus	max	1	0.81	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0	–	–
	min	1	0.75	0.58	0.46	0.36	0.3	0.24	0.2	0.17	0.15	0.13	0.06	0.04

As a result it can be derived that for the calculated values a better accordance is seen for the maximisation than for the minimisation. With variations of more than 100% of the minimal value the calculated values differ remarkably. Comparing the values calculated for the minimisation with those calculated for the maximisation, it can clearly be seen that an unintentional weighting between maximisation and minimisation may be effective.

As an example for the extent of the unintentional weighting between minimisation and maximisation, the values calculated in Table 1 are graphically displayed in Figure 1. It can be observed that for a variation of the optimal value of 100% the value 0 (no effectiveness) is calculated for the maximisation, while a value of 0.5 is calculated for the minimisation, which means that an influence of 50% of the optimal value is still given. In the application of such a formula for the normalisation the decision maker bears a critical responsibility. If such an above-mentioned difference does not comply with the conditions of the decision situation, the calculated result leads to a wrong statement.

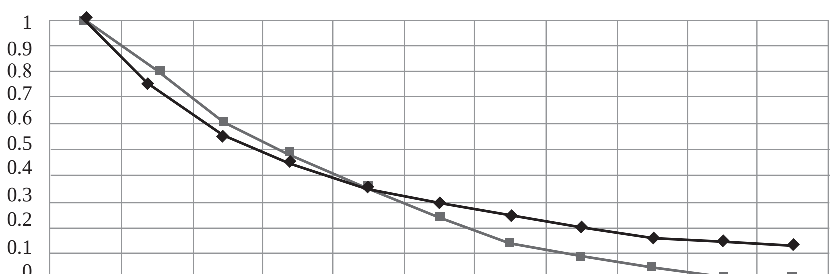


	change	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
min	1.00	0.91	0.83	0.77	0.71	0.67	0.63	0.59	0.56	0.53	0.50
max	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0

Fig. 1. Example for the unintentional weighting between minimisation and maximisation according to formula (4)

With the aim to prevent such an unintentional weighting between minimisation and maximisation formula 5 (Peldschus 1986) was developed. The graphical representation of the calculated values is given in Figure 2. It can clearly be seen that the differences between minimisation and maximisation are considerably smaller. Even for a variation of the optimal value of 100% the difference is only 0.13.

In investigations on the magnitude of these differences no regularity could be observed (Börner 1980). It has not been achieved up to date to develop an ideal formula for this purpose. It is not for nothing that Luce and Raiffa (1967) state "...that this is the Achilles heel of the theory". Oven (1968) proposes a linear transformation under the pre-requisite of a closed interval. Hersh and Carmozza (1976) have found in an empirical study that a



	change	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
min	1	0.75	0.58	0.46	0.36	0.30	0.24	0.20	0.17	0.15	0.50
max	1	0.81	0.64	0.49	0.60	0.50	0.40	0.30	0.20	0.10	0

Fig. 2. Reduction of the unintentional weighting according to formula (5)

utility function does not need to be constant in every case, but can also be represented by a hyperbole function.

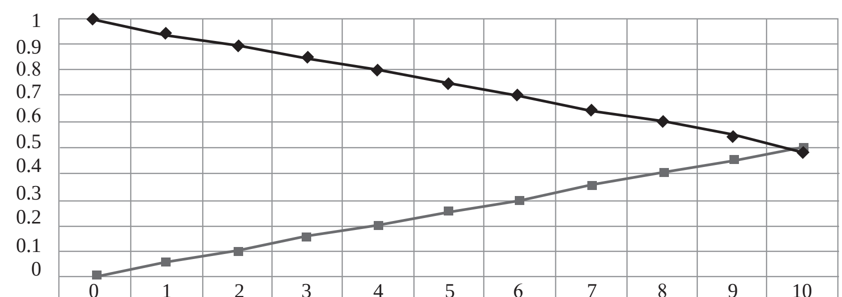
The difficulty is that no generally suitable scaling may be found. Under the condition of a closed range, a corresponding function can be mapped unambiguously to the interval [1; 0]. This is always achieved for the maximisation. In contrast, the range is apparently not limited for the minimisation. This fact is, however, only of theoretical importance. In practical analyses a maximum value, which confines the range, will always exist. Unfortunately, this maximum value is not known in every case and it can therefore not always be implied.

An analysis of Roth (1997) showed that the stability of the solution constitutes a major problem. Within 15 included methods none could be found, for which an inversion of ranks may be excluded. It could only be concluded that for some methods the stability increases with the number of alternatives.

Ahn (1996) concludes that the parameter values may generally occur on every imaginable scale level, if the cardinality of the measurement values is ensured. But as the author also works with interval boundaries, the problem of the stability of the solutions is not resolved thereby.

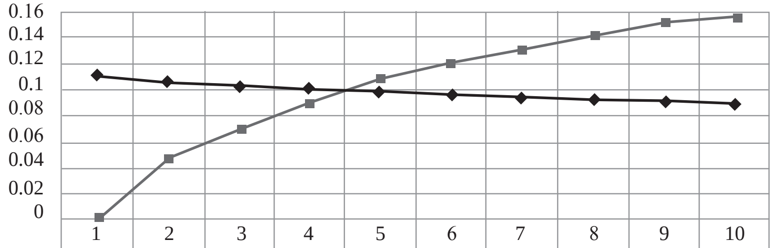
The situation turns out to be somewhat more complex for the square root and the logarithmic function. In these cases the sum of all values becomes effective, and therefore only a tendency can be discussed. Generally speaking, it can be stated that the magnitude of the transformed values depends on the amount of data. If few data are used for the maximisation, the transformed values are greater. The transformed values decrease, if more data are used for the maximisation. For the minimisation, equivalent values are obtained due to the complement to 1 (Fig. 3).

The exemplary values for the logarithmic function are considerably smaller compared to the ones for the square root. It is worth noting that in this representation of the exemplary values contrary curvatures occur between minimisation and maximisation (Fig. 4).



	0	1	2	3	4	5	6	7	8	9	10
min	1	0.949	0.898	0.847	0.796	0.745	0.694	0.643	0.592	0.541	0.49
max	0	0.051	0.102	0.153	0.204	0.255	0.306	0.357	0.408	0.459	0.51

Fig. 3. Example according to formula (6)



	1	2	3	4	5	6	7	8	9	10
min	0.111	0.106	0.103	0.101	0.099	0.098	0.097	0.096	0.095	0.094
max	0	0.046	0.073	0.093	0.108	0.120	0.130	0.139	0.147	0.154

Fig. 4. Example according to formula (7)

3. Solution methods

Concerning the solution methods, a distinction should be made between the orientations towards a game-theoretic equilibrium on the one hand, and methods calculating a rank order on the other hand. The specific conditions for the solution methods and the influence of the calculated characteristic values on the results should be discussed in a separate analysis.

4. Results and conclusions

Linear and nonlinear functions were considered for the calculation of the characteristic values, which describe the problem of the multiple criteria decision problem, and for their transformation to the interval [1; 0] or [1; ~ 0]. It could thereby be concluded that linear functions allow a good mapping to the interval [1; 0]. Problems occur, however, for the minimisation in case characteristic values, which exceed the double minimal value, are included in the description of the variants. In this case other functions need to be used. Nonlinear functions provide an alternative here. It must, however, be noticed that every nonlinear function deforms the original problem. If maximisation and minimisation are jointly required for the solution of the decision problem, the attention should be paid to avoid large differences in the deformation between both cases. A weighting of the importance between minimisation and maximisation is introduced for the solution of the decision problem, if different deformations become effective. If such a weighting cannot be justified, the calculated results must be questioned.

The problem of the different weighting between the objective functions does not occur when considering maximisation goals and minimisation goals separately. A possible error in the transformation of the characteristic values would be the same for all criteria and would thus exert a small influence on the result only.

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DAUGIAKRITERINIŲ SPRENDIMŲ METODAIS GAUTŲ REZULTATŲ KOKYBĖS ANALIZĖ

F. Peldschus

Santrauka

Jau daugelį dešimtmečių susiduriame su daugiakriterinių sprendimų problemomis. Daug metodų iki šiol jau yra sukurta, bet tebekuriami nauji. Dėl metodų įvairovės sudėtinga atlikti išsamią jų apžvalgą. Įvairių metodų palyginimų nėra daug. Kai kuriais atvejais, taikant daugiakriterinius metodus, atliekama daug matematinių operacijų, todėl sunku įvertinti skaičiavimo rezultatus. Dėl šios priežasties galimiems sprendiniams prognozuoti nagrinėjami metodų ypatumai. Atliekama metodų analizė normalizuojant rodiklius intervale [1; 0] arba [1; ~ 0]. Tai linijinė funkcija ir netiesinės (hiperbolinė, kvadratinė ir kubinė, kvadratinės šaknies, logaritminė) funkcijos. Straipsnyje pasiūlyta, kaip sprendimo priėmėjui įvertinti jo sprendimo kokybę.

Reikšminiai žodžiai: daugiakriterinis vertinimas, tiesinė funkcija, netiesinė funkcija, normalizavimas, optimizavimas.

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